New ICMI Study Series

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The Professional Education and Development of Teachers of Mathematics

The 15th ICMI Study

International Commission on Mathematical Instruction

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Contents

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Setting the Stage for the ICMI Study on the Professional Education and Development of Teachers of Mathematics

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The focus of the 15th Study, led by the International Commission on Mathematical Instruction (ICMI), was the professional education and development of mathematics teachers around the world. The study was designed to investigate practices and programs of mathematics teacher education in different countries and to contribute to an international discourse about the professional education of prospective and practicing teachers of mathematics.

The premise of this study was that teachers are key to students' opportunities to learn mathematics. What mathematics teachers know, care about, and do is a product of their experiences and socialization both prior to and after entering teaching, together with the impact of their professional education. This impact is variously significant: in some systems, the effects of professional education appear to be weak or even negligible, whereas other systems are structured to support effective ongoing professional education and instructional improvement.

This study focused on the professional formation of teachers. The curriculum of mathematics teacher preparation varies around the world, both because of different cultures and educational environments and because assumptions about teachers' learning vary. Countries differ also in the educational, social, economic, geographic, and political problems they face, as well as in the resources available to solve these problems. A study focused on mathematics teacher education practice and policy around the world can provide insights useful to examining and strengthening all systems.

We recognize that all countries face challenges in preparing and maintaining a high-quality teaching force of professionals who can teach mathematics effectively and who can help prepare young people for successful adult lives and for participation in the development and progress of society. Systems of teacher education, both initial and continuing, are built on features that are embedded in culture and the organization and nature of schooling. More cross-cultural exchange of knowledge and information about the professional education of teachers of mathematics would be beneficial. Learning about practices and programs around the world can provide important resources for research, theory, practice, and policy in teacher education, locally and globally. The 15th ICMI Study, the Professional Education and Development of Teachers of Mathematics, was designed to offer an opportunity to develop

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a cross-cultural conversation about mathematics teacher education in mathematics around the world.

1. Why Conduct a Study on the Professional Education of Mathematics Teachers?

Three main reasons underlie the decision to launch an ICMI study focused on teacher education. A primary reason rests with teachers' central role in students' learning of mathematics, nonetheless too often overlooked or taken for granted. Concerns about students' learning compel attention to teachers, and to what the *work of teaching* demands, and what teachers know and can do. A second reason is that no effort to improve students' opportunities to learn mathematics can succeed without parallel attention to their *teachers' opportunities for learning*. The professional formation of teachers is a crucial element in the effort to build an effective system of mathematics education. Third, teacher education is a vast enterprise, and although *research on mathematics teacher education* is relatively new, it is also rapidly expanding.

The past three decades have seen substantial increase in scholarship on the education of prospective and practicing mathematics teachers. A growing number of international and national conferences focuses on theoretical and practical problems of teacher education. Publication of peer-reviewed articles, book chapters, and books about the professional education of teachers of mathematics is on the rise. Centers for research and development in teacher education exist increasingly in many settings. A survey team led by Jill Adler reported on the development of research on mathematics teacher education as part of the program at the 10th International Congress on Mathematics Education (ICME-10), in July 2004 in Copenhagen. In addition, it is significant that the past decade also included the launching of a new international journal (in 1996): the *Journal of Mathematics Teacher Education* is published by Springer and edited by an international team of scholars. Furthermore, in contrast with the first milestone International Group of Psychology of Mathematics Education (PME) book (Nesher & Kilpatrick, 1990), which was devoted solely to cognitive research related to student learning of various mathematical topics and concepts, one of the five main research domains of current interest to the PME, as presented in the second milestone PME book (Gutiérrez & Boero, 2006), is the professional life of mathematics teachers and their education.

Mathematics teacher education is a developing field with important contributions to make to practice, policy, theory, and research and design in other fields. Theories of mathematics teachers' learning are still emerging with much yet to know about the knowledge, skills, personal qualities, and sensibilities that teaching mathematics entails and about how such professional resources are acquired. The outcomes of teacher education are mathematics teachers' practice and the effectiveness of that practice in the contexts in which teachers work. Yet we have much to learn about

how to track teachers' knowledge into their practice, where knowledge is used to help students learn. In addition, we have more to understand about how teacher education can be an effective intervention in the complex process of learning to teach mathematics, which is all too often most influenced by teachers' prior experiences as learners or by the contexts of their professional work.

The 15th ICMI Study aimed to assemble from around the world important new work—development, research, theory, and practice—concerning the professional education of prospective and practicing teachers of mathematics. Our goal was to examine what is known in a set of critical areas and what significant questions and problems warrant collective attention. Towards that end, the study aimed to contribute to strengthening the international community of researchers and practitioners of mathematics teacher education, whose collective efforts can help to address problems and develop useful theory.

2. Scope and Focus of the Study

This study focused on the initial and continuing education of teachers of mathematics. We considered the education of teachers at all levels, from those who teach in early schooling to those who teach at secondary schools. Teacher education is a vast topic: this study focused strategically on a small set of core issues relevant to understanding and strengthening teacher education around the world.

The study was organized into two main strands, each representing a critical cluster of challenges for teacher education. In one strand, "Teacher Preparation and the Early Years of Teaching", we investigated how teachers in different countries are recruited and prepared, with a particular focus on how their preparation to teach mathematics is combined with other aspects of professional or general academic education. In this strand, we also invited contributions that offered insight into the early phase of teachers' practice. In the second strand, "Professional Learning for and in Practice", we focused on how the gap between theory and practice is addressed in different countries and programs at all phases of teachers' development. In this strand, we studied alternative approaches for bridging this endemic divide and for supporting teachers' learning in and from practice. This strand was explored across developmental stages—prospective, early years, and continuing practice—of teachers' practice. In both strands, we sought additionally to learn how teachers in different countries learn the mathematics they need for their work as teachers and how challenges of teaching in a multi-cultural society are addressed within the professional learning opportunities of teachers.

Table 1 provides a graphic representation of the scope and focus of the study. The table makes plain that for Strand 1, the focus was only on the prospective and early years of teaching; the study did not focus on issues of recruitment, program structure, and curriculum for practicing teachers. However, Strand II focused on professional learning in and from practice at all phases of teachers' development.

Table 1 Scope and focus of the study

	Phases of teacher development	
	Initial teacher education (prospective and early years of teaching)	Continuing practice
Strand I: Programs of teacher education (recruitment, structure, curriculum, first years)	Yes	No
Strand II: Professional learning for and in practice	Yes	Yes

2a. Strand I: Teacher Preparation Programs and the Early Years of Teaching

This strand of the study focused on a small set of important questions about the initial preparation and support of teachers in countries around the world at the prospective stage and into the early years of teaching. How those phases are structured and experienced varies across countries, as does the effectiveness of those varying structures. Questions central to the investigation of initial teacher preparation and beginning teaching included:

1. Structure of teacher preparation: How is the preparation of teachers organized—into what kinds of institutions, over what period of time, and with what connections with other university or collegiate study? Who teaches teachers, and what qualifies them to do so? How long is teacher preparation, and how is it distributed between formal study and field or apprenticeship experience? How is the preparation of teachers for secondary schooling distinguished from that of teachers for the primary and middle levels of schooling?
2. Recruitment and retention: Who enters teaching, and what are the incentives or disincentives to choose teaching as a career in particular settings? What proportion of those who prepare to teach actually end up teaching and for how long? How do teachers' salaries and benefits relate to those of other occupations?
3. Curriculum of teacher preparation: What are specific central challenges for the curriculum of teacher preparation? How do different systems experience, recognize, and address these issues? Is interdisciplinarity in teacher education commonplace, and if so, how is it managed?
4. The early years of teaching: What are the conditions for beginning teachers of mathematics in particular settings? What supports exist, for what aspects of the early years of teaching, and how effective are they? What are the special problems faced by beginning teachers, and how are these experienced, mediated, or solved? What is the retention rate of beginning teachers, and what factors seem to affect whether beginning teachers remain in teaching? What systems of evaluation of beginning teachers are used, and what are their effects?
5. Most pressing problems of preparing teachers: Across the initial preparation and early years, what are special problems of teaching mathematics within a

particular context, and how are beginning teachers prepared to deal with these problems?

6. History and change in teacher preparation: How has mathematics teacher preparation evolved in particular countries? What was its earliest inception, and how and why did it change? What led to the current structure and features, and how does its history shape the contemporary context and structure of teacher education?

2b. Strand II: Professional Learning for and in Practice

This strand of the study added substantive focus in complement to the first. Whereas the first strand examined programs and practices for beginning teachers' learning, the focus of the second related to teachers' learning across the lifespan. This strand's central focus is rooted in two related and persistent challenges of teacher education. One problem is the role of experience in learning to teach; a second is the divide between formal knowledge and practice. Both problems led to the central question of Strand II: how can teachers learn for practice in and from practice?

Although most teachers report they learned to teach "from experience", researchers and practitioners alike know that experience is not always a good teacher. Prospective teachers enter formal professional education with many ideas about good mathematics teaching formed from their experiences as pupils. Their experiences in learning mathematics have often left them with powerful images of how mathematics is taught and learned as well as who is good at mathematics and who is not. These formative experiences have also shaped what they know of and about the subject. These experiences, along with many others, affect teachers' identities, knowledge, and visions of practice in ways which do not always help them teach mathematics to students.

Moreover, the education of prospective and practicing mathematics teachers often seems remote from the work of teaching mathematics and does not necessarily draw on or connect to teachers' practice. Opportunities to learn from practice are not the norm in many settings. Teachers may of course sometimes learn on their own from studying their students' work; they may at times work with colleagues to design lessons, revise curriculum materials, develop assessments, or analyze students' progress. In some countries and settings, such opportunities are more than happy coincidence; they are deliberately planned. In some settings, teachers' work is structured to support learning from practice. Teachers may work with artifacts of practice—videotapes, students' work, curriculum materials—or they may directly observe and discuss one another's work. We sought to learn about the forms such work can effectively take and what the challenges are in deploying them.

Strand II of the study asked how mathematics teachers' learning may be better structured to support learning in and from professional practice at the beginning of teachers' learning, during the early years of their work, and as they become more experienced. Central questions include:

1. What sorts of learning seem to emerge from the study of practice? What do teachers learn from different opportunities to work on practice—their own or others'? In what ways are teachers learning more about mathematics, about students' learning of mathematics, and about the teaching of mathematics as they work on records or experiences in practice? What seems to support the learning of content? In what ways are teachers learning about diversity, about culture, and about ways to address the important problems that derive from social and cultural differences in particular countries and settings?
2. In what ways are practices of teaching and learning mathematics made available for study? How is practice made visible and accessible for teachers to study it alone or with others? How is "practice" captured or engaged by teachers as they work on learning in and from practice (e.g., video, journals, lesson study, joint research, observing one another and taking notes)?
3. What kinds of collaboration are practiced in different countries? How are teachers organized in schools (e.g., in departments) and what forms of professional interaction and joint work are engaged, supported, or used?
4. What kinds of leadership help support teachers' learning from the practice of mathematics teaching? Are there roles that help make the study of practice more productive? Who plays such roles, and what do they do? What contributions do such people make to teachers' learning from practice?
5. What are crucial practices of learning from practice? What are the skills and practices, the resources and the structures that support teachers' examination of practice? How have ideas such as "reflection", "lesson study," and analysis of student work been developed in different settings? What do such ideas mean in actual settings, and what do they involve in action?
6. How does language play a role in learning from practice? What sort of language for discussing teaching and learning mathematics—professional language—is developed among teachers as they work on practice?

3. Design of the Study

The Study on the Professional Education of Teachers of Mathematics was designed to enable researchers and practitioners around the world to learn about how teachers of mathematics are initially prepared and how their early professional practice is organized in different countries. In addition, the study took aim at an endemic problem of professional education, that is, how learning from experience can be supported at different points in a teacher's career and under different circumstances.

Towards this end, the first phase of the study was the dissemination of a discussion document announcing the study and inviting contributions. The discussion document defined the focus of the study on the two main strands of interest—Teacher Preparation and the Early Years of Teaching and Professional Learning for and in Practice—and invited proposals for participation in a study conference. We welcomed individual as well as group proposals, focusing on work within a single program or setting as well as comparative inquiries across programs and settings.

In order to make grounded investigations of practice in different countries possible, we invited proposals in three formats: papers, demonstrations, and interactive work sessions. Papers were intended to report on analysis of practices and programs of mathematics teacher education in particular settings, with attention to the main questions and foci of the study. We invited research reports, conceptual-analytic or theoretical papers grounded in examples of practice, and descriptions, accompanied by evidence appropriate to the claims of the paper. Demonstrations were intended to make as vivid as possible materials, approaches, or practices to enable careful examination and critical discussions. Interactive work sessions were intended to offer for a group of researchers and practitioners attending the conference the opportunity to work on a common problem of mathematics teacher education.

The second phase of the study was the study conference, held in Brazil from 15–21 May 2005, bringing together 147 researchers and practitioners from around the world. As is the normal practice for ICMI studies, participation in the study conference was by invitation only, given on the basis of a submitted contribution. We received an unprecedented number of proposals of papers, demonstrations, and interactive work sessions for the study conference, making decisions about who to invite difficult. As a consequence, we ended up with a larger number of invited participants than the originally planned 120, making this study conference the largest of all past ICMI study conferences. We chose proposals from diverse researchers and practitioners who could make solid practical and scientific contributions to the study: researchers in the field, those actively engaged in curriculum development for the education of prospective and practicing teachers in various settings, and mathematicians who play a crucial role in preparing and supporting teachers who are not specialists of the discipline. To ensure a rich and varied scope of resources for the study, participants from countries under-represented in mathematics education research meetings were invited.

The conference was deliberately designed for active inquiry into the professional education of teachers of mathematics in different countries and settings. To take full advantage of the opportunity, there were no oral presentations of papers during the conference. Some sessions offered a critical commentary of the papers accepted, discussions of the papers in small interactive groups, and the extraction of key issues and synthesis. Other sessions included interactive demonstrations or work sessions. In addition to the above activities there were plenary sessions that included panels and presentations as well as sessions for cross-strand communication of ideas and issues.

The publication of this study volume—a report of the study's achievements, products, and results—is the third phase of the study.

4. The Study Volume

The study volume aims to assemble from around the world important new work—development, research, theory, and practice—concerning the professional education of teachers of mathematics. Our goal is to examine what is known in a set of critical areas and what significant questions and problems warrant collective attention.

The study volume is designed to represent the issues worked on before and during the study conference in a way that captures their complexity, detail, and subtlety and to represent and include the unusual breadth of participation in this study (nationally, types of people in terms of where or what they work on in teacher development, levels of experience). The study volume is based on selected contributions and reports prepared for the conference as well as on the outcomes of the conference. We aimed to include various contributions from conference participants, but the book does not simply reprint each individual contribution. This was unfeasible because of the sheer quantity but also we wanted to use what was worked on at the conference, not just what each person brought to it. Thus, while revised versions of some papers are included, in other cases, examples were developed based on particular papers. Some of the book also synthesizes themes developed within the strand groups. The editorial board decided to be as inclusive as possible and to give a voice to all interested conference participants. Thus, instead of aiming at a conventional format, in which we select authors and invite them to write different chapters, we designed a unique plan that gives a voice to every conference participant who wanted to contribute. As a consequence, sometimes a large number of people contributed to one chapter.

Two main sections of the volume focus on the main strands of the study, which are, respectively, "Initial Mathematics Teacher Education" and "Learning in and from Practice". Each of these sections includes:

- An articulation of key issues or problems in each of the main strands of the study: initial teacher preparation around the world and learning in and from practice.
- Examples of programs or practices that address those issues or problems and what is known or needs to be examined about their effectiveness.
- Commentaries on these key issues and on the nature of programs or practices examined in this study related to those issues; questions raised by what we learned about these issues and the programs or practices related to them.
- Resources for use in the education of prospective and practicing teachers of mathematics.

The editor of the section "Initial Mathematics Teacher Education" is Stephen Lerman. This section includes three themes: The preparation of teachers, edited by Jarmila Novotná; Student teachers' experiences and early years of teaching, edited by Stephen Lerman; and Mathematics educators' activities and knowledge, edited by Pedro Gómez. Invited by the editors of this volume, comments and reflections, addressing the section as a whole, were written by Gilah Leder and Shiqi Li.

The editor of the section "Learning in and from Practice" is Barbara Jaworski. This section includes four themes: Development of teaching in and from practice, edited by Barbara Jaworski; Mathematics teachers' professional development: Learning in and from practice, edited by João Filipe Matos; Tools and settings supporting mathematics teachers' learning in and from practice, edited by Marja van den Heuvel-Panhuizen; and The balance of teacher knowledge: Mathematics and pedagogy, edited by Terry Wood. Invited by the editors of this volume, comments and reflections, addressing the section as a whole, were written by Aline Robert and Chris Breen.

To spur research in this field, the section "Key Issues for Research in the Education and Professional Development of Teachers of Mathematics" presents the thinking of four key people about major problems of practice and policy, the questions that are crucial to ask, how these might be investigated productively, and what such investigation would take. In this section Ubiratan D'Ambrosio reflects on the purposes of education and on the role of mathematics teachers as educators; Edward A. Silver addresses the problem of practice-based professional development for mathematics teachers; and Jill Adler and Barbara Jaworski present a collaborative view on the state of research on mathematics teacher education and how it needs to develop.

The concluding section includes commentary. We hope that this volume will be useful to the mathematics education community as well as to other researchers, practitioners, and policy makers concerned with the professional education of teachers.

Section 1
Initial Mathematics Teacher Education

Editor: **Stephen Lerman,** *London South Bank University, London, UK*

The following is slightly modified from the proposal for the study group:

In this section of the 15th study group we examined a set of important questions about the initial preparation and support of teachers in countries around the world, at the pre-service stage, and into the early years of teaching. How those phases are structured and experienced varies across countries, as does the effectiveness of those varying structures. Questions central to the investigation of initial teacher preparation and beginning teaching included:

a. The structure of teacher preparation: How is the preparation of teachers organized—into what kinds of institutions, over what period of time, and with what connections with other university or collegiate study? Who teaches teachers, and what qualifies them to do so? How long is teacher preparation, and how is it distributed between formal study and field or apprenticeship experience? How is the preparation of teachers for secondary schooling distinguished from that of teachers for the primary and middle levels of schooling?
b. Recruitment and retention: Who enters teaching, and what are the incentives or disincentives to choose teaching as a career in particular settings? What proportion of those who prepare to teach actually end up teaching and for how long? How do teachers' salaries and benefits relate to those of other occupations?
c. Curriculum of teacher preparation: the study sought to probe a small set of key challenges of the teacher preparation curriculum and investigate whether and how different systems experience, recognize, and address these issues. Two such issues are:

   - What is the nature of the diversity that is most pressing within a particular context—for example, linguistic, cultural, socio-economic, religious, racial—and how are teachers prepared to teach the diversity of students they will face in their classes?
   - How are teachers prepared to know mathematics for teaching? What are the special problems of subject-matter preparation in different settings, and how are they addressed? Is interdisciplinarity in teacher education commonplace, and if so, how is it managed? How do faculty in education interact with faculty in mathematics over issues of teacher education?

In addition, we received proposals that identified and examined other specific central challenges for the curriculum of teacher preparation.

d. The early years of teaching: What are the conditions for beginning teachers of mathematics in particular settings? What supports exist, for what aspects of the early years of teaching, and how effective are they? What are the special problems faced by beginning teachers, and how are these experienced, mediated, or solved? What is the retention rate of beginning teachers, and what factors seem to affect whether beginning teachers remain in teaching? What systems of evaluation of beginning teachers are used, and what are their effects?
e. Most pressing problems of preparing teachers: across the initial preparation and early years, what are special problems of teaching mathematics within a particular context, and how are beginning teachers prepared to deal with these problems?
f. History and change in teacher preparation: How has mathematics teacher preparation evolved in particular countries? What was its earliest inception, and how and why did it change? What led to the current structure and features, and how does its history shape the contemporary context and structure of teacher education?

A substantial number of papers were submitted, and the working groups took these papers and worked with the ideas and research evidence presented. Summaries were prepared and discussed at plenary sessions at the end of the conference. The writing of this section, therefore, both represents the papers submitted and the discussions and work that followed. Contributors are acknowledged in each chapter where their contribution to the conference was used in the writing.

We have divided the report into three themes, each of which has its own introduction in which the chapters are described. The first theme, the preparation of teachers, edited by Jarmila Novotná, addresses issues in initial teacher education, a and c from the list above. The second theme, edited by Stephen Lerman, looks at student teachers' experiences on their teaching practice/practicum and in their first years of teaching, addressing d and e above. Questions b and f are, to some extent, dealt with in these themes. We found a number of papers addressing the work of mathematics-teacher educators and hence the third theme, edited by Pedro Gómez, is entitled "Mathematics educators' activities and knowledge".

The literature on mathematics teacher education, going back to the early 1980s if not earlier, has highlighted the importance of initial mathematics teacher education in broadening and expanding prospective teachers' horizons on the process of learning, on the nature of mathematical activity, and on the range of strategies available to teachers. At the same time the literature points out how hard it is to succeed in this programme. Most student teachers come with a view of what it is to teach mathematics, dominated by their own learning experiences and their decision to take up mathematics teaching often explained in terms of their ability to explain mathematics well. Whilst this is an important component of mathematics teaching, it is certainly not the only component. The evidence shows that initial mathematics teacher education is too often less effective than we would wish. It is therefore vital that we examine and study what we do and its effect or lack of effect on our student teachers in the hope that we can do it better.

Theme 1.1
The Preparation of Teachers

Editor: **Jarmila Novotná,** *Charles University, Prague, Czech Republic*

The teacher's task is to enable his or her students to develop their individually different processes of knowledge building and meaning construction as well as positive attitudes (De Corte, 2000). It is a common belief that mathematics is a difficult subject. Therefore, in order to help learners succeed it is of the utmost importance that the teacher should examine and analyse possible barriers that might have a negative impact on learning. A good mathematics teacher should be able to suggest ways to minimize these and to use a variety of effective teaching strategies that help to overcome individual learning difficulties.

The general question of Theme 1 is "What professional skills, what attitudes are to be acquired for the teaching of mathematics?" Learning to teach (not only on the pre-service level) requires a balance between teachers' theoretical and practical knowledge and skills including knowledge of mathematics, knowledge of teaching mathematics, and knowledge of psychology and pedagogy. These components are only general; they do not answer the basic question about the content and extent of the knowledge required from future teachers.

The theme is introduced by a survey on pre-service[1] teacher education, "Overview of teacher education systems across the world," written by Tatto, Lerman, & Novotná. The survey is based primarily on two sources: contributions to the plenary panel at the 15th ICMI study conference, coordinated by M. T. Tatto (panelists J. Novotná, D. Tirosh, & R. Spanneberg), "Framing the questions: Understanding mathematics teacher education cross-nationally"; and individual descriptions of mathematics pre-service teacher training systems delivered by Strand 1 participants and summarized by S. Lerman.

The body of the theme is organized in Chapters 1.1.1, 1.1.2, and 1.1.3. The first of them could be roughly characterized as "what?" and the second and third as "how?" In another perspective we could say that the first chapter represents "theory", the second "practice", and the third "application".

In the first chapter, "Components of mathematics teacher training", Liljedahl summarizes the main ideas concerning the acquisition of knowledge required for the teaching of mathematics. The text was discussed with J. Hodgen, A. Peter-Koop, &

[1] In-service teacher education is dealt with in another part of the book.

J. Novotná, but the final writing was by P. Liljedahl. The following domains are discussed: knowledge and beliefs for the teaching of mathematics and structures of and research in initial teacher education.

In the second chapter, "Practising mathematics teacher education: Expanding the realm of possibilities", Gellert presents a well-organized set of stimulating examples from the practice of teacher education in various countries and (teacher) education systems. This diversity has two aims: to offer interesting and often non-standard examples of "best practices", as well as to start discussion about their role in mathematics teacher training. Examples are grouped in four domains: activation of understanding school mathematics, improvement of communication of mathematics ideas, use of information and communication technology in mathematics teacher training, and study of classroom practice. The examples have the form of commented original texts from the contributions to the study conference.

In the final chapter, "Learning to teach mathematics: Expanding the role of practicum as an integrated part of a teacher education programme", Bergsten, Favilli, and Grevholm deal with teaching-practice organization, cooperation with schools, and its connection to theoretical courses. They use the term "practicum" for teaching practice within an institutionalised education programme. It means here the work of a student teacher as a practising teacher in a school, under the supervision of an experienced teacher. The text is illustrated by examples presented at the study conference.

The program of the study conference dealing with initial teacher training was much broader than the issues presented here. The limited length of the chapter does not allow presenting all of them. We invite readers to read the chapters in the conference proceedings.

Chapter 1.1.1
Overview of Teacher Education Systems Across the World

Maria Teresa Tatto, *Michigan State University, Michigan, USA,*
Stephen Lerman, *London South Bank University, London, England, UK,* and
Jarmila Novotná, *Charles University, Prague*

From a system as well as from an institutional perspective, the education of teachers is dependent on the occurrence of a number of interconnected events which are in turn closely related with the life cycle of teachers' careers: entry to teacher education signals the beginning of teachers' careers, and it comes accompanied by recruitment and selection processes and by individual expectations among those choosing to become teachers. The knowledge acquired during teacher education is expected to transfer into knowledge for teaching and, presumably, on improved pupil learning. In all instances teacher education evolves within and interacts with social, economic, and political contexts.

The following text is based on two sources: contributions to the plenary panel at the 15th study conference, coordinated by M. T. Tatto (panelists J. Novotná, D. Tirosh, and R. Spanneberg), "Framing the questions: Understanding mathematics teacher education cross-nationally"; and individual descriptions of mathematics pre-service teacher-training systems delivered by Strand I participants and summarized by S. Lerman.[1] National contributions to this overview are listed in the references but not within the text.

The characterization of teacher education systems across the world and the opportunities they provide teachers in learning about mathematics is shaped by the following components:

- the entry to the profession (e.g., characteristics of future teachers and what they bring with them);
- the processes of learning to teach (e.g., the structure and approach followed by teacher education programmes and the programmes' curricular sequence); and
- the outcomes of such learning experiences (e.g., the knowledge teachers acquire, including the approaches they learn to teach).

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[1] Particular characteristics of the teacher education systems summarized in this chapter will be published as an article in a specialized research journal.

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The text is framed by the following questions describing the framework of the 15th study conference plenary panel:

- What are the system characteristics?
- What are the institutional characteristics? (Who goes into teaching? Who are the teacher educators? What is in a credential? What is in the curriculum?)
- What are the structure and approaches to teacher education? (What is the overall structure of the curriculum? What content goes into the teacher education curriculum? What are the links between theory and practice?)

1. System Characteristics

Current trends in teacher education reform reflect a need for unification (e.g., in the European Union the Bologna Declaration) as well as a need to attend to local demands as a response to compete in a global economy (see Tatto, 2007a, b). The response to these pressures is diverse. Whereas in some country contexts the need to compete in the economy has prompted teacher education programmes to add more years and to provide deeper content knowledge, thus relocating teacher preparation in universities, in others the trend is towards more emphasis on training on the job. A review of the materials provided by the ICMI data illustrates the diverse trends.

Among the twenty country regions[2] meeting at the study conference, many reported the universities as the major context for teacher education. Whereas in some country contexts this has been a long-standing tradition (e.g., Germany), others, such as South Africa, have just recently moved in this direction. In other country contexts teacher education is located in national teacher colleges or it represents a combination of both, education in universities and in teacher colleges (e.g., pedagogical institutes for elementary teachers). England's teacher education presents the most diversity, as it is located both in universities and in schools and it also permits a number of variations, including an examination-only option. Uganda covers the most range by including education in universities, local and national teacher colleges, and schools, and in-service education is offered via the distance education model.

Another important characteristic of contemporary teacher education is its level of regulation. Most of the ICMI participants reported some kind of regulation at the national level (usually via ministries of education) and state or local levels (via local ministries). In sum there seems to be a trend towards increased regulation whether located at the national or at the local levels. The United Kingdom and France illustrate a strong regulatory and centralized system.

[2] We refer here to country regions or contexts rather than full countries, as ICMI participants reported that in some cases data on this and in other aspects of teacher education is not available nationally; thus, the information we have available represents the best approximation to what we know at this point. Readers should note that the data was collected in mid-2005 and some changes may have occurred.

Systemic change in teacher education is enacted in institutions, including teacher education programmes and schools. In the following section we explore in broad strokes how the institutions offering teacher education are organized across different settings. Questions of interest relate to recruitment and selection, the structure of teacher education programmes, the content of the curriculum, and the credentials obtained.

2. Institutional Characteristics

From an institutional perspective, entry to teacher education may or may not be selective, but even when entry is non-restrictive, personal choices (or self-selection) are at work. Who enters the profession is not only determined by institutional selection policies and programme requirements, but also by the personal choices individuals make regarding the programmes they enter and the courses they take. Institutions that deliberately recruit and select future teachers may be assumed to use rationales based on "what works" locally or globally. Relevant characteristics, such as entry-level qualifications, years of study, years of tertiary schooling, and years of experience, gender, and age have been considered to be predictors of effective teaching practices and pupil achievement in studies on school effects on mathematics knowledge in several countries. There is great variability in the kinds of requirements, length, and quality of professional education offered to teachers across contexts. Most participants reported that entry level to the profession occurs after secondary education, with various entry-level selectivity criteria.

Intrinsically linked to structural characteristics, teacher educators can be seen as most important in shaping the teacher education curriculum. Those teaching teachers are differentiated according to their institutional departments and disciplines. Thus for the most part mathematicians teach mathematics courses, and in some cases these are taught by mathematics educators who may have a mathematics degree as well. For the most part pedagogy courses are taught by educators or in some cases mathematics educators who may have backgrounds in psychology, sociology, or philosophy or may be experienced teachers. Practicum is for the most part supervised by practicing teachers and teacher educators.

Another important characteristic of teacher education institutions is the awarding of credentials and the point in teachers' lifecycles when this occurs. The information gathered via the ICMI participants as well as from the international literature signals a growing trend to award credentials to teachers at least at the first stage of tertiary education. In some cases mathematics teachers teaching higher levels hold credentials at the second level of tertiary education. As previously discussed, as important as the credential is, it provides limited information regarding the level of knowledge attained by teachers and whether such knowledge provides the basis for effective teaching. Understanding what constitutes a credential amounts to looking into the "black box" of teacher education and into the opportunities to learn designed to support teacher knowledge. Again we resort to the international literature to uncover these connections. Most prospective mathematics teachers reach the first

stage of tertiary education (International Standard Classification of Education, 1997, ISCED 5A). Others obtain the second stage of tertiary education, leading to an advanced research qualification (ISCED 6), as some countries seem to require a full master's qualification before entering the profession.

3. Structure and Approaches to Teacher Education

Teacher education characteristics (e.g., programme structure) and approach or orientation (e.g., curricular content and sequence, pedagogy) importantly shape opportunities to learn and may influence teachers' knowledge, practice, and presumably pupil learning. The information obtained at the study conference shows great variability among the options offered future mathematics teachers. Nevertheless, pre-service teacher education can be generally understood as concurrent (bringing together general education and pedagogy plus teaching practice and/or practicum); consecutive (general preparation occurs independently of teacher education and also includes teaching practice and/or practicum). In-service preparation may include any of the options described previously.

For the purposes of this report, *concurrent preparation* is defined as the joint occurrence of general education and professional education in a single programme. In addition this definition includes varying periods of field-based practice or practicum. In the case of the 15th study conference, programmes included preparation on the mathematics content and pedagogy and varied lengths of field experience or practicum. The length of all these periods was quite variable, and for the most part the period for both mathematics and pedagogy preparation ranged from three to six years, and the period of practice also varied from eighty days to a year.

For the purposes of this report *consecutive preparation* consists of an independent period of general education and a separate period of teacher preparation, to which a varying period of practice may follow. In the case of the 15th study conference reports, the length of programmes varied from two to five years of general education (e.g., in mathematics) and from one to four years of teacher preparation. Periods of practice ranged from forty-five days to two years.

An important part of the curriculum sequence and delivery can be found on the teaching practice and practicum components. There is great variability in this area, with some programmes providing only limited practice and others providing extensive periods of partially independent practice. Still, a further question concerns the curriculum emphasis in different approaches to teacher education. The ICMI study provides more information in this area.

As in other areas of teacher education approaches, it should not be surprising to find a great deal of variety in the curriculum offered to teachers within and across countries. However, this is an area in which the field suffers from definitional problems wherein labels may mean a variety of things (e.g., what is the content included in a mathematics pedagogy course?). We take as standard the different dimensions of teachers' professional knowledge (content knowledge, pedagogical content knowledge, pedagogy, knowledge of pupils, and knowledge of context) as

proposed by Shulman (1987) as a beginning point and recognizing that these still remain for the most part theoretical propositions. Participants' reports indicated whether teachers who would eventually teach mathematics after their preparation were specialists, the proportion of time dedicated to the study of the mathematics content, the mathematics pedagogy content, and/or the pedagogy content.

3a. Emphasis on Mathematics Content

The information provided by conference participants shows varied degrees of emphasis on opportunities to learn related to mathematics content. Most primary teachers are educated as generalists and most of the preparation they receive places low emphasis on mathematics content (as per the proportion of time dedicated to mathematics courses as part of their overall programme). The lack of emphasis on mathematics in primary teacher education programmes may be attenuated by those programmes' selection strategies (e.g., requiring a high level of mathematics knowledge) or by those programmes that are consecutive, thus ensuring that future teachers bring a high level of mathematics knowledge to their teacher preparation (see previous section on the structure of teacher preparation). In some cases, however, teachers graduate from programmes with little or no knowledge of mathematics. This situation also applies to the preparation of secondary teachers. However, the fact that many of these are not specialists presents yet another possible troubling trend regarding the level of mathematics knowledge teachers may hold.

3b. Emphasis on Mathematics Pedagogy

Similarly, the information gathered from ICMI participants shows varied degrees of emphasis on opportunities to learn related to mathematics pedagogy content. The study conference participants' reports on this area reveal (with some exceptions) that those who design teacher education programmes give some degree of emphasis to what we have called here "pedagogical content knowledge" in mathematics. The trend shows a higher emphasis given to this knowledge in the education of secondary teachers, while a possible troubling trend that would need to be explored further is the lack of emphasis given to pedagogical content knowledge in the education of primary teachers.

3c. Emphasis on Pedagogy

General pedagogy seems to be a major if not the only emphasis in most teacher education programmes. In contrast with the information on content pedagogy discussed previously, the report on pedagogy emphasis is more consistent. Pedagogy is given a high level of emphasis in the preparation of primary teachers and a possible troubling trend towards low emphasis in the preparation of secondary teachers.

3d. *Practicum Experiences*

The notion that future teachers need to have the opportunity to practice what they learn seems to be widely recognized in the field of mathematics teacher education. According to the information provided by the participants, practically every programme makes allowances for some kind of field experience. The information shows a trend that seems to signal a tendency to longer periods of practicum as an essential part of mathematics teacher preparation, thus presumably strengthening the links between theory and practice. This presumption, however, is still subject to empirical investigation.

4. Open Questions for Future Research

Important questions that remain unanswered, at least in the mostly descriptive studies, are: What are the differential effects of teacher education approaches to facilitating teachers' graduation, hiring, and permanence in the profession? What are the differential effects of teacher education approaches on the mathematics knowledge that prospective teachers acquire? What are the effects of different teacher preparation arrangements on mathematics teaching? How is pupil achievement affected by those teachers who have received diverse types of teacher preparation versus those who have not received teacher preparation? How is the teaching force affected by those teachers who enter and remain in the profession versus those who do not? For new programmes dedicating more time and resources to reach the ambitious goals set by current educational reforms, it becomes even more relevant to know the answers to these questions. The lack of research-based answers has implications for future empirical research on mathematics teacher education. Drawing from our reading of the literature and from the information provided by conference participants, we suggest possible directions for future inquiry:

1. There is a need for sound research on the characterization of teacher education systems and the paths through which they are likely to influence teacher knowledge, teacher practice, and pupil learning. This is especially true regarding issues dealing with diversity and notions on how people become good teachers, in the curricular emphasis placed on different aspects of teacher education (mathematical knowledge, pedagogical knowledge, practical knowledge), and the diverse methods for doing research on teacher education (research organized in small-scale vs. large-scale studies).
2. These studies would need to be comparative (across and within countries) and would need to better conceptualize and define the constructs and indicators of the intended cognitive and pedagogic influences of teacher education on teacher knowledge and practice moving away from reducing findings and highlighting the influence of context on these experiences.
3. It is essential to begin to explore the efficiency of teacher education and the implications of policy borrowing within and across contexts.

Examples of new questions include: What are the learning opportunities that high-quality mathematics teachers of highly performing pupils have? How are these opportunities different across contexts and diverse pupils? What are the systemic characteristics that produce and sustain these practices?

In sum, as our limited data gathering indicates, there is a growing need to design policy-oriented studies according to a typology of comparative differences within and across regions that can give better insights on the teacher education-teacher practice-pupil learning continuum, taking into account contextual differences. The framework to understand teacher education systems may include the following distinctive features: questions about the educational goals for individual learning; particular societies' ideals of an educated individual; approaches to learning, school, and classroom strategies; the level at the educational system at which these models seem to place more emphasis; countries' and systems' administrative styles; level of centralization with which the system is organized; unit costs; and financial sources. The framework may draw on similarities such as the expected processes and outcomes of teacher education. The use of a comparative framework would help educators and policy makers understand not only which characteristics of different education systems seem to have an important impact on teacher education, teaching practice, and pupil learning, but also the systemic conditions that make it possible.[3] Understanding these conditions is essential in conceptualizing viable policy.

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Chapter 1.1.2
Components of Mathematics Teacher Training

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1. Introduction

Initial mathematics teacher education is primarily concerned with knowledge—the acquisition of knowledge required for the teaching of mathematics. Opinions as to what exactly comprises this knowledge and how it is best delivered and best learned varies widely across different contexts. In what follows, we will look more closely at this concept of teacher knowledge and how it plays itself out in the context of initial mathematics teacher education.

2. Knowledge and Beliefs for the Teaching of Mathematics

Teacher knowledge is most often discussed as being comprised of three strands: content knowledge, pedagogical knowledge, and didactical knowledge (Durand-Guerrier & Winsløw, 2005). Shulman (1987) refers to these same categories, respectively, as subject matter knowledge (SMK), pedagogical knowledge (PK), and pedagogical content knowledge (PCK). In the context of mathematics education, content knowledge pertains to mathematical concepts, use of mathematical techniques, mathematical reasoning, proof, and so forth. PK is subject independent and deals with general principles of education such as theories of learning; sociological, psychological, and ethical aspects of education and its functions (Durand-Guerrier & Winsløw, 2005); and classroom management and assessment. Didactical knowledge is the knowledge regarding the conditions and ways of mathematics teaching and learning (Bloch, 2005; Brousseau, 1997; Durand-Guerrier & Winsløw, 2005) and "captures both the link and the distinction between knowing something for oneself and being able to enable others to know it" (Rowland, Thwaites, & Huckstep, 2005). In general, the three strands can be seen as knowing the mathematics, knowing teaching, and knowing how to teach mathematics.

This is not to say that this is the only way in which teacher knowledge can be partitioned. Bergsten and Grevholm (2005) speak of teacher knowledge as being

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comprised of disciplinary knowledge and PK. Disciplinary knowledge is the substantive knowledge of facts, procedures, concepts, and so forth, as well as knowledge of mathematics as a discipline. PK, on the other hand, is PCK and curriculum knowledge as well as knowledge of general issues in education, such as learning, developmental psychology, and socialisation. Adler and Davis (2005) view the acquisition of teacher knowledge as learning to teach and learning mathematics for teaching. Rowland et al. (2005) introduce the notion of the Knowledge Quartet, which is "a tool for thinking about the ways that a teacher's subject knowledge comes into play in the classroom" (p. 2). This quartet is comprised of "foundation (teachers' knowledge, beliefs, and understandings acquired "in the academy"), transformation (teachers' knowledge in action as demonstrated both in planning to teach and in the act of teaching itself), connection (binds together certain choices and decisions that are made for the more or less discrete parts of mathematics education), and contingency (witnessed in classroom events that are almost impossible to prepare for)" (paraphrased from Rowland et al., 2005, p. 2) .

Variation and further partitioning of each of the aforementioned knowledge strands allow for more fine-grain discussion of the particularities of teachers' knowledge. Didactical knowledge, for example, has been extensively elaborated on to account for the specific knowledge that is needed for the teaching of specific mathematical concepts (García & Sánchez, 2005). Such elaborations start with a topic and work outwards to encompass specific strategies, tasks, and assessment instruments that will facilitate the learning of that topic. An altogether different elaboration of didactical knowledge is task knowledge (Liljedahl, Chernoff, & Zazkis, 2007), which refers to teachers' knowledge of the mathematical and pedagogical affordances that exist within a given mathematical task.

With respect to PK, constructivism in its many forms (Bruner, 1966; Dewey, 1916; Piaget, 1951; Wertsch, 1985) are still foundational elements of most initial teacher education programmes. However, more recently conceived theories are also starting to have a presence in these programmes. Situated learning (Lave & Wenger, 1991), for example, and its discourse on communities of practice as it pertains to both the classroom context and the teacher education context is beginning to have an influence on teacher education. This discourse is re-casting what it means to be a participant in these communities (see García & Sánchez, 2005). Although not as overtly present in teacher education curriculum as constructivism and situated learning, a number of contemporary theories are having local influences on what is imparted as PK in various initial teacher education programmes. Imaginative education (Egan, 2005), with its descriptive emphasis on cognitive tools and prescriptive emphasis on capitalizing on students' propensity for accessing particular cognitive tools at different developmental stages, is recasting what it means for a learner to develop. Likewise, theories on learning as communicating (Sfard, 2001) and classrooms as complex (learning) systems (Davis & Simmt, 2003) are beginning to take hold in some initial teacher education contexts (see Proulx, 2005).

The content knowledge considered relevant to teacher education is also not immune to the variance of context. Although the broad brush strokes of curriculum

have not changed much over recent decades, the finer details show greater variability through time. Subsequently, the specific content knowledge required for the teaching of mathematics has also changed. Surprisingly, however, the content knowledge that is required of teachers hasn't changed much. This is primarily due to adherence to the traditions of mathematics teacher education and the structures they have entrenched. This will be discussed in greater details in the Structures of Initial Mathematics Teacher Education section, which follows.

However, discussions of teachers' knowledge cannot be strictly limited to these objective forms—teachers' subjective knowledge is also important. "It has become an accepted view that it is the [mathematics] teacher's subjective school-related knowledge that determines for the most part what happens in the classroom" (Chapman 2002, p. 177). One central aspect of subjective knowledge is beliefs (Op 'T Eynde, De Corte, & Verschaffel, 2002). In fact, Ernest (1989) suggests that beliefs are the primary regulators for mathematics teachers' professional behaviour in the classrooms. These beliefs do not develop within the practice of teaching, however.

Prospective elementary teachers do not come to teacher education believing that they know nothing about teaching mathematics (Feiman-Nemser, McDiarmid, Melnick, & Parker, 1987). "Long before they enrol in their first education course or math methods course, they have developed a web of interconnected ideas about mathematics, about teaching and learning mathematics, and about schools" (Ball, 1988). These ideas are more than just feelings or fleeting notions about mathematics and mathematics teaching. During their time as students of mathematics they first formulated, and then concretized, deep-seated beliefs about mathematics and what it means to learn and teach mathematics (Lortie, 1975). These beliefs often form the foundation on which they will eventually build their own practice as teachers of mathematics (Skott, 2001). Unfortunately, these deep-seated beliefs often run counter to contemporary research on what constitutes good practice. As such, it is the role of teacher education programmes to reshape these beliefs and correct misconceptions that could impede effective teaching in mathematics (Green, 1971).

This distinction between knowledge and beliefs is a false dichotomy, however. At the level of teachers' action this distinction is not so clear. In general, knowledge is seen as "essentially a social construct" (Op 'T Eynde, De Corte, & Verschaffel, 2002). That is, the division between knowledge and belief is the evaluation of these notions against some socially shared criteria. If the truth criterion is satisfied then the conception is deemed to be knowledge. However, when teachers operate on their knowledge the distinction between what is true and what they believe to be true is not made. Leatham (2006) articulates this argument nicely:

> Of all the things we believe, there are some things that we "just believe" and other things we "more than believe—we know." Those things we "more than believe" we refer to as knowledge and those things we "just believe" we refer to as beliefs. Thus beliefs and knowledge can profitably be viewed as complementary subsets of the things we believe (p. 92).

3. Structures of Initial Teacher Education

Initial teacher education is largely aimed at developing an integrated proficiency with the aforementioned forms of knowledge and beliefs. How this is achieved varies greatly across different contexts. Our goal here is not to go into the minutiae of these differences, but rather to highlight some of the general commonalities. First, however, we introduce some terminology to create distinction between teachers in their varying stages of development. In particular, we are interested in drawing a distinction between what we refer to as prospective teachers and pre-service teachers. Pre-service teachers are teacher candidates working within a teacher education programme. Prospective teachers, on the other hand, are individuals who have decided that they would like to become teachers and have begun the process of acquiring some of the necessary prerequisite knowledge and/or experiences (in the form of courses and/or required volunteer experience) to be accepted into a teacher education programme. We make this distinction because both the nature of their experiences and their expectations of these experiences are very different.

In general, initial teacher education is separated into generalist teacher education and specialized mathematics teacher education. This divide is most often facilitated along a divide between elementary teacher education and secondary mathematics teacher education.[1] Elementary teacher education programmes, for the most part, require very little mathematical content knowledge of their prospective teachers. The knowledge they do require, however, is usually very specific and consists primarily of elementary mathematics content knowledge (Flowers, Rubenstein, Grant, & Kline, 2005). At the same time, these programmes may require their candidates to acquire some PK, either of the general form (e.g., psychology of learning) or the specialized form (e.g., learning disabilities). Secondary mathematics teacher education programmes, on the other hand, often require highly specialized mathematics content knowledge. The nature of this knowledge is quite different, however. Whereas prospective elementary teachers are required to obtain mathematical knowledge relevant to the teaching of elementary mathematics, prospective secondary teachers are often required to obtain mathematical knowledge that is of a more academic nature (Moreira & David, 2005; Opolot-Okurut, 2005). That is, they are required to become proficient in the university-level mathematics taught to a wide spectrum of mathematics, engineering, and science students. Other than this requirement of university-level mathematical knowledge there are rarely any other requirements placed on them.

This discrepancy between what is required of elementary teachers and secondary mathematics teachers is primarily due to a confluence of pragmatics and tradition. Elementary teachers are, for the most part, generalist teachers having to be proficient in the teaching of all subjects. Thus, to require prospective elementary teachers to obtain a large amount of university-level content knowledge in all of the

[1] The exception to this is in settings where elementary mathematics is taught by a specialist mathematics teacher.

school subjects prior to entering a teacher education programme is rather unrealistic. Instead, these programmes opt for proficiency with school-level content knowledge in all of the subject areas as a requirement.

Such pragmatics does not extend to the requirements of prospective secondary teachers, however. As already mentioned, prospective secondary mathematics teachers are often required to obtain mathematical content knowledge of a form that is not obviously relevant to secondary mathematics. The reason for this is tradition—the tradition of what it means to be a mathematics teacher. Since the classical period, to be a mathematics teacher meant that one was first a mathematician. This thinking has changed very little in the last 2,500 years. Prospective mathematics teachers must first become mathematicians.

Once accepted into a teacher education programme, pre-service teachers of both a generalist (elementary) and a specialized (secondary) type are subjected to very similar experiences. Through courses and seminars their repertoire of content knowledge, PK, and didactical knowledge is expanded, and through case studies and practicum experience these discrete forms of knowledge are integrated (for more information, see Subtheme 3).

Teacher education is a unique enterprise. The reason for this is that the *what* is also the *how*. That is, what we teach is also how we teach. As such, pre-service teachers have a unique experience. What they are learning is also how they are learning. Through their experiences as student teachers they are both student and teacher, and through the constant shifting between student and teacher they are given the opportunity to not only acquire the knowledge that they will require to become effective teachers, but also are given the opportunity to recast their initial (pre-conceived) beliefs about what it means to be a teacher, what it means to teach, what it means to learn, and even what it means for something to be mathematics. Through this recasting process they begin to form an identity of who they are as a teacher, and what it is that they teach as a subject.

4. Research in Initial Teacher Education: Past, Present, and Future

Research in initial teacher education is vast, contributing to all its aspects from prospective teachers' initial beliefs (Liljedahl, 2005) to teacher identity (Lerman, 2001) to the effectiveness of a specific teacher education method (Chapman, 2005). This research can be viewed in terms of two dimensions: how it contributes to the domain of knowledge that teachers need for teaching and how best to help teachers acquire this knowledge. In keeping with the recurring theme of knowledge for teaching in this chapter, we will focus only on the first of these.

The domain of knowledge that teachers need for teaching possesses a duality within mathematics education—a duality that can be encapsulated as the tension between "has" and "should have". That is, there is a constant tension in the literature between the knowledge that a teacher "has" and the knowledge that a teacher

"should have". In many ways this is a product of the constant confluence of theory, research, and practice within the field of mathematics education and cannot be, and should not be, resolved by the exclusion of one or the other. Our understanding of what knowledge is needed for the teaching of a specific mathematical concept is informed by the knowledge possessed by teachers who are effectively (or not effectively) teaching that concept (Ball, Hill, & Bass, 2005). This emerging understanding, in turn, informs our work in pre-service and in-service teacher education as we work to develop the necessary knowledge within teachers.

For example, research into students' misconceptions and analysis of student thinking (Flowers et al., 2005) about specific mathematics concepts can be seen as a informing the discourse of what teachers need to know in order to teach those specific concepts. Also informing this discourse is research into teachers' practice of teaching these same concepts. The results of both of these forms of research coupled with mathematical analysis of students' understanding and/or teachers' practice contributes greatly to the didactical knowledge base.

Similarly, research into prospective and teachers' knowledge of subject matter helps to extend the discourse on content knowledge. However, this extension of discourse should be viewed more as a focusing rather than an expansion. For example, extensive work has been done on pre-service teachers' understanding of elementary number theory (see Zazkis & Campbell, 1996). The fine-grain analysis associated with this research has alerted us to subtle variations and the developmental nature of pre-service teachers' understanding of this content. As such, it focuses our own understanding of this particular SMK, as well as gives insights into what and how to teach elementary number theory to pre-service teachers.

5. Concluding Remarks

Initial teacher education is primarily concerned with developing proficiency with a number of different dimensions of teacher knowledge, from teachers' knowledge of mathematical content to teachers' knowledge of pedagogy and didactics. Although much of initial teacher education deals with these different dimensions discretely, a significant portion is often devoted to treating these dimensions in an integrated manner. As pre-service teachers progress through the initial teacher education experience, these different forms of knowledge are wound tighter and tighter together until the content of their experience can best be described as knowledge needed for teaching. That is, initial teacher education can be viewed as the beginning of a braid (see **Fig. 1.1.2.1**). At the beginning stages the different dimensions of teacher knowledge are represented by individual and discrete strands. As teacher education progresses, these strands are braided together to form a tighter experience in which, although still distinguishable from one another, the different strands are integrated. In ideal circumstances this braid tightens towards the end of the initial teacher experience to form a unified fibre, the content of which is teacher knowledge.

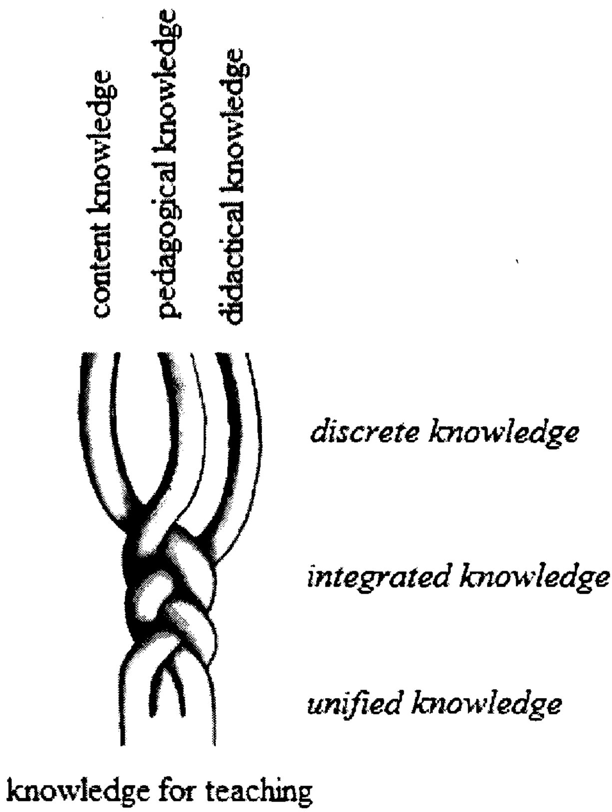

Fig. 1.1.2.1 Teacher knowledge in initial teacher education

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Chapter 1.1.3
Practising Mathematics Teacher Education: Expanding The Realm of Possibilities

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1. Introduction

It is often said that student teachers' underlying beliefs of what mathematics consists of and how it should be taught are restricted in two ways. On the one hand, future elementary teachers in general use only weak mathematical conceptions, which often do not help them to realise their educational ambitions. On a general educational level, many of these students advocate discovery learning and collective problem solving, but when it comes down to the mathematical activities that have to be prepared, their experience of "traditional" school mathematics is of little help. On the other hand, future (higher) secondary teachers mostly are very well prepared with respect to formal academic mathematics when entering mathematics education programmes, either because they have already passed a mathematical formation at university or because their teacher education programmes emphasise the study of academic mathematics and not of educational or didactical modules. Being socialised as mathematicians, and not as mathematics teachers, these future teachers often lack the experience of how to convert formal mathematics into school mathematical activities.

For both future teachers, elementary as well as secondary, building conceptions of mathematically rich and cognitively and socially stimulating school mathematical activities is at the heart of the process of their professional formation. Mathematics teacher education, in that sense, provides opportunities for future mathematics teachers to expand the realm of their possibilities.

However, the title of this chapter carries a second meaning. By presenting examples from the practice of teacher education, we aim at expanding the realm of possibilities for and within programmes of mathematics teacher education. These possibilities can be seen, again, as activities: by adapting and transforming the diverse examples presented here, teacher educators may organise new and different activities for future teachers to actively develop their professional knowledge.

In line with this view, this chapter does not intend to propagate "best practices". "Best practices" strive towards generating "perfect teachers", thus reflecting a technocratic cause-effect mentality, or a "training" mindset, in the project of achieving

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the perfect image of a mathematics teacher. However, the outcomes of any mathematics teacher education programme, or of single courses and activities therein, are much more diverse and unpredictable than might be expected: the perceptions, interpretations, and uses are going to be different for each student teacher (Proulx, 2005).

This chapter, instead, tries to widen the horizon of programmes and activities in mathematics teacher education by presenting stimulating examples from diverse countries and teacher education cultures. First, these examples may directly contribute to an enrichment of mathematics teacher education practices. Second, and reflectively, the diversity of the examples presented here may provoke a reconsideration of the objectives of the mathematics teacher education programmes in use.

The examples from teacher education practice to be presented in this chapter have been grouped into four areas, thus reflecting their main purposes for the education of future mathematics teachers:

- Activating the understanding of school mathematics
- Enhancing the communication of mathematical ideas
- Using information and communication technology (ICT) in mathematics teacher education
- Studying classroom practice

While this chapter draws to some extent from the existing body of literature about teacher education practices, the examples displayed in boxes were all presented and discussed at the 15th ICMI Study. The boxes are excerpts from the study conference papers. This chapter does not scrutinise these examples analytically but, instead, offers a bouquet of activities and experiences, thus trying to fire the reader's imagination.

It should be noted that within the respective conference papers most of these examples have been discussed and used within a research context. This research context is rather ignored in the chapter on hand.

2. Activating the Understanding of School Mathematics

School mathematics can be regarded as an autonomous body of knowledge. It is not a simplistic form of academic mathematics. It is not striving exclusively for symbolic abstraction and rigour. In order to be meaningful for the majority of the students, it tries to construct visual representations for mathematical concepts and relations. Whereas, for instance, academic mathematics defines mathematical concepts symbolically and tries to avoid redundant formulation, school mathematical knowledge of a mathematical concept comprises the diverse representations of the concept as well as the translations between them.

One focus of mathematics teacher education practices is to activate the student teachers' understanding of school mathematics by involving them in school mathematical activities of translation between different representations. This proves to

be useful for both future primary school teachers (Amato, 2005; Gadanidis & Namukasa, 2005; Peretz, 2006) and future secondary-school teachers (Bloch, 2005).

Amato (2004, 2005) reports that although student teachers generally correctly perform the multiplication of large numbers, only a few of them use the concept of place value to explain why numbers move over in the partial products:

> The student teachers were asked to explain the reason for leaving blank the units' place of the second addend in the multiplication algorithm for 45 × 123. The most frequent type of explanation was related to place value, but it did not involve much conceptual understanding: "Because I am now working with the tens' place, then I write the next number under the tens". Eight student teachers wrote about calculating 4 times 123 and none wrote about calculating 40 times 123. Two student teachers said it was to make the result bigger. The ideas presented by a few student teachers were thought to have the potential to develop the belief that mathematics is an irrational subject: "I would say that the place is reserved for the + [addition] sign" (Amato, 2005).

Amato uses school children's activities as a strategy to activate student teachers' understanding of school mathematics, as shown in the following example:

Versatile representations (Amato, 2004), like the area representation for multiplication, were used in activities in order to represent together two or more related concepts and operations and so to make their relationships clear. The student teachers were first given some practice in using the area representation with concrete materials [a plane version of Dienes blocks, Fig. 1.1.3.1] and later they were asked to interpret and draw area diagrams [Fig. 1.1.3.2].

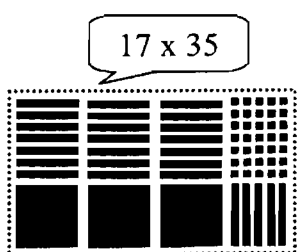


Fig. 1.1.3.1 Plane version of Dienes blocks

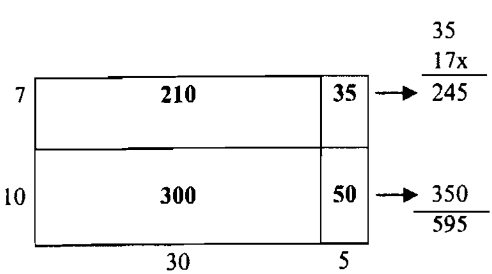


Fig. 1.1.3.2 Area diagram

From her experience with this kind of activity Amato concludes that the student teachers' understanding of the area representation of multiplication is related to an explicit teaching of the conventions used in those representations. Accordingly, she introduces more activities for multiplication of two-digit numbers:

> In order to help student teachers understand the conventions used in the area representation, an analogy was made with constructing a wall with big bricks (hundreds), medium bricks (tens) and small bricks (units). The student teachers were asked which they thought it would be quicker to construct a wall: (a) to use as many bigger bricks as possible or (b) to start the construction by using small bricks? Before using the "bricks" to construct the wall they were asked to use the strips (tens) and little squares (units) as "rulers" to measure the base and height of the wall [**Fig. 1.1.3.3**]. After they finished constructing the wall they were asked to remove the rulers and verbalise the four partial multiplication sums (7×5, 7×30, 10×5, 10×30).

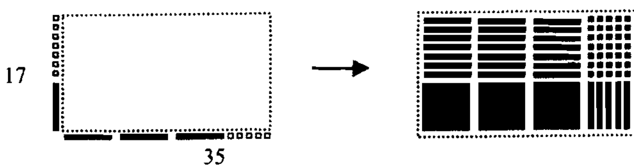
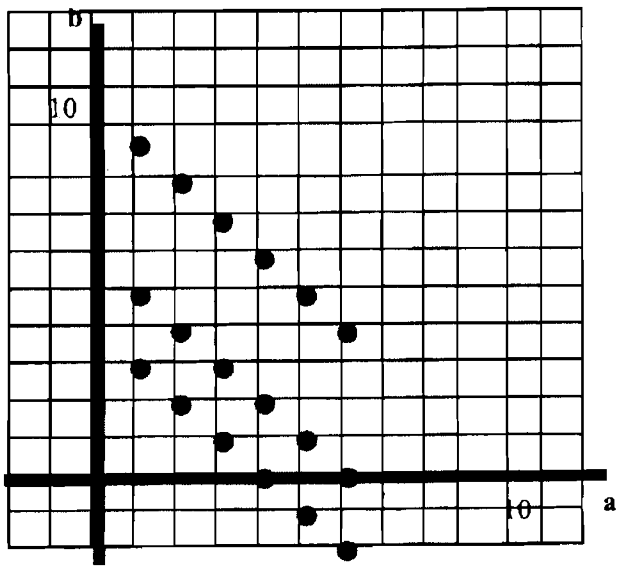


Fig. 1.1.3.3 Base and height of the wall

Gadanidis and Namukasa (2005) provide another example of how primary-school student teachers can get involved in school children's mathematical activities. They design activities to be "interesting and challenging enough to capture their [future primary teachers] interest and imagination and to offer the potential for mathematical insight and surprise." They use a variety of school mathematics problems and situations for exploration, all being problem-solving tasks that were non-routine to the student teachers, for example:

> One of the problems explored the equation $\_+\_= 10$. Pre-service teachers rolled a die to get the first number and then calculated the second number. They wrote the pairs of numbers in table and in ordered pair form, and plotted the ordered pairs on a grid. We repeated this for $\_+\_= 6$ and $\_+\_= 4$. Some pre-service teachers expressed surprise that the ordered pairs lined up [**Fig. 1.1.3.4**]. "I had the 'aha' feeling when I saw the diagonal line pattern on the graph. That was my favourite part." Pre-service teachers also noticed that the graph of $\_+\_= 4$ could be used as a visual proof of $6 + (-2) = 4$ and $5 + (-1) = 4$. That is, $(6,-2)$ and $(5,-1)$ line up with $(4,0)$, $(3,1)$, $(2,2)$ and

(1,3). They also explored equations whose graphs were not parallel to the ones in [**Fig. 1.1.3.4**] and whose graphs were not straight lines. Such mathematical connections appeared to be pleasing to the pre-service teachers. "I loved the adding/graphing we did and how you should take problems and branch out... it really makes something in my mind click."



a + b = 10, a + b = 6 and a + b = 4

Fig. 1.1.3.4 Ordered pairs lined up

While it may appear as a straightforward method to involve future primary-school teachers in activities that on the one hand intend to activate their understanding of school mathematics and, on the other hand, may serve as a blueprint for future teaching of primary-school mathematics, this is not obviously the case for novice secondary mathematics teachers. In many countries, these future teachers have received a mathematical formation similar to that of a mathematician before entering courses in mathematics education. Bloch (2005) points to the fact that students "often get a very formal conception of mathematics during their university courses. For them, a theorem has to get a proof, but no justification in terms of problem solving, it is seen as a part of a mathematical theory which its own justification." This socialisation into academic mathematics is completed by a very specific way of knowledge transfer. Through their mathematical formation, future secondary mathematics teachers get used to the idea that mathematics teaching has to be done by

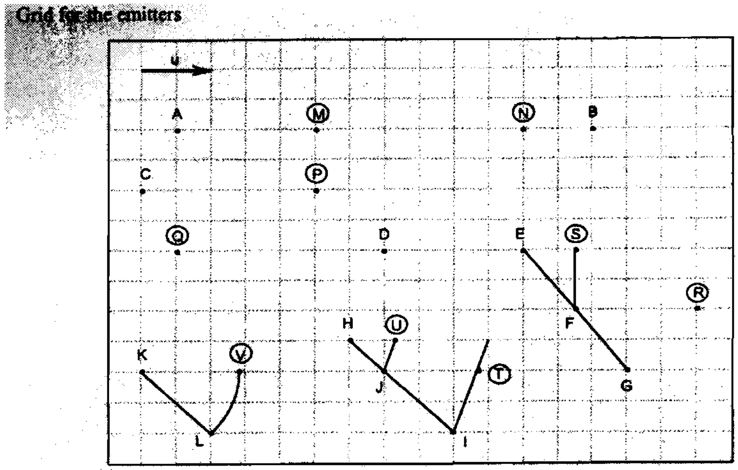
a teacher in front of the students and that the teacher tells the mathematical laws and explains mathematical algorithms. As Bloch (2005) observes, student teachers "have no idea that the mathematical law could be understood, overall, considering that only elementary mathematics are in question at that level. The mathematical formalism seems transparent to them. They are accustomed to take what the mathematics teacher said at University for granted and cannot imagine any other behaviour from the students in their own classes."

From this observation, Bloch draws the necessity to offer new situations to make their knowledge of school mathematics evolve also to the novice secondary teachers and new activities to get them to know what mathematical interactions with their students are. For example, Bloch introduces a "grid game":

A situation to introduce the product of vectors by real numbers has been tested with novice teachers. It consists of a communication about collinear vectors and decomposition in a basis, whose support is a grid. The direct game simply consists in calculating sums of vectors, and associating them to the correct points, as usually done. This first direct game institutes a heuristic milieu, the milieu where students can get the technique and the basic strategy: they discover that if they multiply a vector by a number they can start from a point and reach another point. The type of instruction at this phase is: let A be a point of the plane, V a given vector; place the point B such as AB = V.

The inverse game has got two phases itself: In Phase 1 the game aims to find points by doing the product of one given vector by numbers. What is at stake in this Phase 1 is the way, how students relate real numbers and lines in the plane. Students work in groups in which there are two emitters and two recipients. Emitters—who dispose of a schema with points that are unknown to the recipients—have to send a message to their corresponding recipients to make them find the unknown points [see **Fig. 1.1.3.5**].

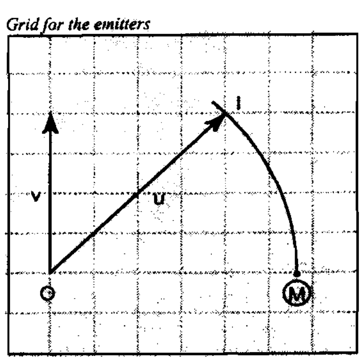
The second phase works with the functionality of a two vectors basis in the plane. It is a communication game too, but in a two dimensional system (a basis). In Phase 2 students have to find that, two non colinear vectors and a point being given, by sum and product, one can reach unknown points [see **Figs. 1.1.3.6** and **1.1.3.7**]. If reaching every point is not effectively possible, restraining to integer coefficients is not enough to understand the generality of the rule: the students have to do the calculation in some non trivial cases. The main objective is to make students understand the rule of how a vector basis operates, before they are told the formal expression of this rule. For future teachers, the situation has the objective of understanding by action that with a basis of two vectors they can reach every point of the affine plane; this is a pragmatic proof of the functionality of the concept of basis; and, it makes student teachers discover that pragmatic proofs are not evident even when a formal proof is well-known. For that purpose, it is necessary to let young teachers effectively reach some points with real coefficients or rational numbers (Bloch, 2005).



The other team has got the same grid as you, but with only the points A to L and the vector u. Send them messages to place the points M to V. You're not allowed to tell geometric descriptions in your messages, that must contain only well known points, u and numbers.

Fig. 1.1.3.5 Phase 1

Insightful experience of school mathematics is an important starting point for the development of teachers' perceptions and classroom practice. These examples react on the difficulties primary and secondary mathematics teachers face when introducing standard mathematical concepts and procedures in the classroom. They



The other team has got the same grid as you, but with only the point O and the vectors u and v. Send them a message to place the point M. It is on the circle (O, OI), and on a straight line orthogonal to v. But you are not allowed to tell it in your message that must contain only O, u, v and numbers.

Fig. 1.1.3.6 Phase 2, with circle

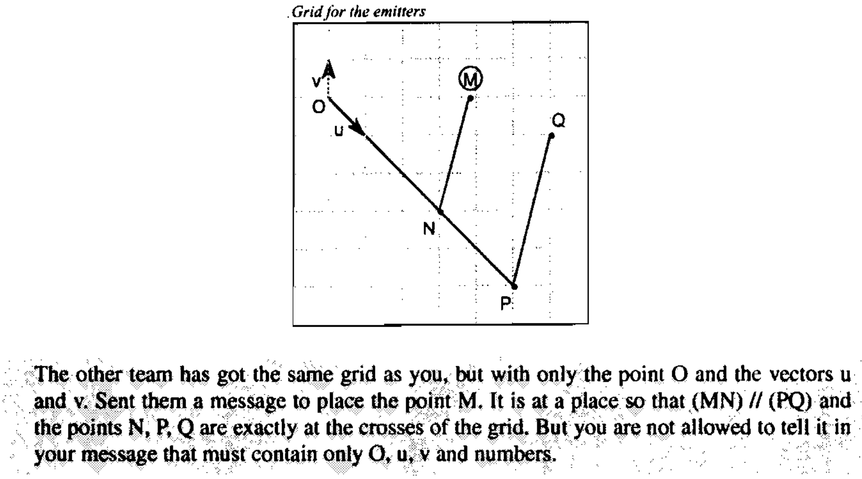


Fig. 1.1.3.7 Phase 2, with parallels

demonstrate that school mathematical activities, which in all three examples are essentially related to the representational character of school mathematics, need to be regarded as an important pillar of any teacher education programme.

3. Enhancing the Communication of Mathematical Ideas

The community of researchers in mathematics education broadly accepts the claim that language matters in the mathematics classroom. Many facets of this issue have been investigated (e.g., Adler, 2001; Alrø & Skovsmose, 2002; Brown, 1997; Cobb & Bauersfeld, 1995; Kieran, Forman, & Sfard, 2002; Pimm, 1987). However, with respect to the preparation of mathematics teachers, this important issue is frequently neglected or, not much better, treated implicitly, based on the conviction that a language to express mathematical ideas develops automatically from mathematical activity. This point of view is theoretically naïve with respect to the pupils' learning of mathematics. It is counterproductive with respect to future teachers learning to teach mathematics. Both Grevholm and Bergsten (2005) and Peter-Koop and Wollring (2005) point to the fact that the development of a mathematics teacher's professional language is a critical issue for mathematics teacher education. Pre-service teachers should be aware that the creation of a mathematical language is an essential part of mathematical activities.

This creation, indeed, is not a simple and automatically occurring phenomenon. According to Bernstein (1996) everyday knowledge is context dependent and segmentally organised and consistent within each segment, but segments overlap and knowledge organisations often do not match. The same holds for everyday activities and everyday language. In contrast, school mathematics is systematically principled

and hierarchically organised. It requires the development of a mathematically consistent language. It can be argued that this problematic issue should be explicitly reflected within teacher education practice. This paragraph shows two examples of explicit enhancing of the communication of mathematical ideas and concepts in mathematics teacher education.

Peter-Koop and Wollring (2005) refer to common classroom experience as well as to psychological research when arguing that "especially with young children, (mathematics related) action seems to precede the ability to express one's own (mathematics related) thoughts, ideas and strategies in words". They conclude that the development and the use of non-verbal language, which reflect mathematics-related actions, are substantial for improving the process of teaching and learning. This is particularly important for "the communication about shape, number and structure in the primary mathematics classroom. From our point of view, mathematics yields special communication platforms with respect to functional communication—especially through the exploration of various iconic forms of articulation and communication, such as children's drawings or folding posters". Peter-Koop and Wollring demonstrate that future teachers as well as primary school pupils can successfully develop (mostly) non-verbal instructions for folding a special paper star:

> The example illustrates how different forms of iconic articulation can support the communication of mathematical ideas. The context of the two examples below is the exploration of geometrical concepts through paper folding activities. The pictures show two "folding posters" (in German: "Faltplakate") designed by fourth graders [**Fig. 1.1.3.8**] and student teachers from a geometry course [**Fig. 1.1.3.9**]. The idea was to provide a folding instruction for second graders that would work without further oral explanation. The two

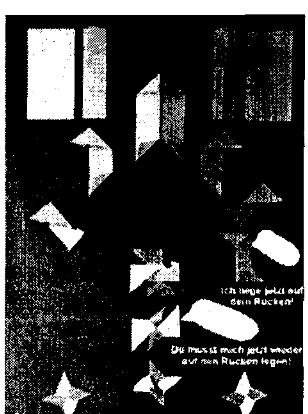
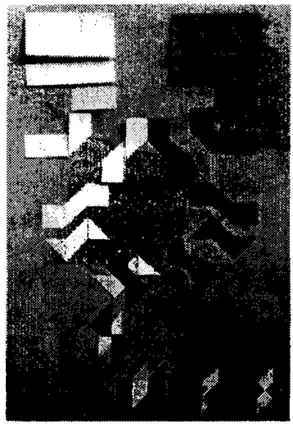


Fig. 1.1.3.8 Faltplakat designed by fourth graders



Translation of the pupils' comments in Figure 1.1.3 h: "Now I am lying on my back!" "You have to put me on my back again!"

Fig. 1.1.3.9 Faltplakat designed by student teachers

folding posters illustrate in how far the fourth graders as well as the student teachers communicate key construction ideas which are related to the special properties of the object—in this case a star that is made out of two congruent but symmetrically opposite parts (Peter-Koop & Wollring, 2005).

Grevholm and Bergsten (2005) report on a development project in mathematics teacher education that aims at explicitly developing a mathematics teachers' professional language. In their project, groups of four to five student teachers work collaboratively, and without interference from teacher educators, with core concepts in mathematics. These future teachers are asked to produce a shared and agreed result in written form. The results from the various groups are compared, and thereafter a videotape from one group's collaborate activity is jointly discussed. The focus of this reflection is on the language that is used to explore and explain the mathematical tasks. According to Grevholm and Bergsten (2005), it is highly important to create open, explorative tasks with focus on core concepts of school mathematics "that promote mathematical discussion and reasoning in the group work of student teachers". An example of such a mathematical task, with the core concepts of mode, median, and average, is presented below:

Does the director tell the truth about salaries?
The director Birger Jonasson in the ICT-corporation High-Tec is interviewed on TV and talks about the fact that there is a high salary level of the company.

The thirteen employed have an average salary of 166555 SEK per month. The mode (called "typvärde," typical value in Swedish) for a monthly salary is 1 million SEK.
The reporter asks how big the median salary is.
"Yes, it is 16000 SEK per month but that is not so interesting in this connection," claims the director.
Question 1
Does the director tell the truth? Can the facts given about the salaries really be true? What could the salary pattern look like?
Question 2
Three different statistical measures are mentioned in the text. When is one and when the other measure relevant to use? How did the director choose to measure and why do you think he did so?
Question 3
How would you plan a teaching sequence about statistical measures for pupils in Years 5 and 9 respectively? Make a draft plan that you think is good and explain why you have chosen this model. What knowledge about statistical measures do you consider important for the pupils?
Question 4
What did you learn from this exercise? How does it differ from earlier tasks that you have solved about statistical measures? Can pupils in compulsory school solve this type of tasks? Do you find such tasks in the textbooks?
Material
Text from the terminology book *Matematikterminologi i skolan*, quotations from the compulsory school mathematics curriculum and mathematics textbooks for years 4–9 (Grevholm & Bergsten, 2005).

4. Using Information and Communication Technology in Mathematics Teacher Education

Information and communication technology (ICT) is used within mathematics teacher education courses for three purposes, which can be separated analytically. First, ICT is a means to facilitate student teachers in learning to teach mathematics. For instance, student teachers can be offered access to databanks of videotaped classroom interaction or to platforms for joining "virtual communities" (Bairral & Zanette, 2005). Second, ICT can be treated as a teacher's tool to teach mathematics to schoolchildren. Third, ICT can be regarded as being an integral part of school mathematics. In this last sense, it is an aim of mathematics teacher education to accustom student teachers to the view that innovative mathematics education is ICT based: "ICT is not just a simple auxiliary tool. It is an essential technological element that shapes the social environment, including mathematics teaching.

Therefore, it influences the mathematics teacher's evolution regarding professional knowledge and identity" (da Ponte, Oliveira, & Varandas, 2002, p. 113). In the praxis of mathematics teacher education the three approaches are not strictly separated. Accordingly, Miller (2005), who is making use of metaphors from the fields of ecology and anthropology, speaks of an "ICT-rich mathematical education environment" and an "ICT culture". He starts from the premise that in order to meet the ICT standards set by official (governmental or state) regulations pre-service teachers should be introduced into ICT-rich mathematical education environments. Miller (2005) provides a description of a rather extensive and comprehensive ICT environment in a British university, in which pre-service teachers learn to teach ICT-based school mathematics.

An ICT-rich mathematical-education environment

"Students are quick to adopt the current climate of opinion about the role of new technologies that they see exemplified by their own subject teachers." To this end the mathematics course design not only allows the teacher educators to act as role models but also looks to set an ICT culture providing:

- over 90% of the sessions where the use of an interactive whiteboard is central to learning
- ICT applications such as spreadsheets, graph plotters and geometry programmes that are integrated "seamlessly" into sessions
- regular timetabled subject-based ICT sessions
- a requirement that all students will have used ICT at least twice with a class or small group of pupils on the first 7-week teaching practice
- a subject-based ICT assignment
- a form to monitor the student's use of ICT with pupils
- a laptop computer for a group of students to use in school with pupils
- CD-ROM of resources that includes ICT training resources
- interactive whiteboard specific software, user guides and a tutorial programme that can be followed to learn how to use an interactive whiteboard
- lesson materials that incorporate ICT and/or interactive whiteboard use

To assure that all our students can demonstrate that they have ICT as a "strength" we have linked aspects of ICT knowledge, understanding and skills with the formal assessment process (Miller, 2005).

Among these conditions for ICT-rich environments in mathematics teacher education, Miller (2005) identifies four features of ICT in mathematics courses that are key to the British situation: basic support for cooperative work, the use of the interactive whiteboard, spreadsheet skills, and the use of ICT in the teaching of school mathematics:

Basic support for cooperative work

We require that our students share work and ideas, so we have placed them in eight groups. Within these groups they have to: solve a mathematical problem and produce a solution using, for example, PowerPoint; design a number of posters to illustrate the mathematics possible in a topic, such as trees or jewellery; produce a resource and lesson plan related to citizenship in mathematics; and provide a review of a website useful for the teaching and learning of mathematics. To facilitate sharing we use a free-access website, Basic Support for Cooperative Work (BSCW: http://bscw.fit.fraunhofer.de/), where any individual can register, create an area, invite others to join, and allow users to upload and download files. The assessment process requires that students have to put their poster, citizenship, and website information onto the shared area.

The use of the interactive whiteboard

The majority of our mathematics students are placed in at least one school where they will be able to use an interactive whiteboard. We therefore require, as part of the assessment process, that they produce an original interactive whiteboard resource, of at least 10 pages, for a one-hour lesson together with an appropriate lesson plan, and place both parts on the BSCW site. The software is on the CD-ROM, and we provide sessions on how to use it.

Spreadsheet skills

At the start of the course all the mathematics students already have reasonable personal spreadsheet skills. Therefore to challenge them we require that they make an interactive worksheet in Excel. This is a file that provides an activity for pupils and then offers feedback. Typically these files use: "if" conditions, to set up automatic checking of answers; conditional formatting, allowing automated coloured responses; "macros", to clear answers and set up new questions; and scroll bars, to change parameters. Once completed the interactive worksheet has to be tried in school with pupils. This then has to be written up, with the report and file placed on the BSCW site. Almost always students report success with the interactive worksheet, indicating that it motivates pupils and supports understanding. Occasionally students show pupils how to make such a worksheet and ask them to design one for younger pupils. A similar activity involves the use of some commercial short programmes with pupils.

The use of ICT in the teaching of mathematics

At two specific points in the course we ask students to report on their use of ICT with pupils. This involves a lesson plan, the ICT resource, any other materials used in the lesson and, most importantly, a critical review and evaluation of the lesson plan and the lesson. All these components have to be placed on the BSCW site. One session comes at the end of the first teaching practice and the other midway through the second practice, so we expect there to be a clear change of understanding as students become more experienced both generally and in terms of ICT use (Miller, 2005).

Miller concludes that pre-service teachers need substantial guidance both on the "technical" side of ICT-rich school mathematics and on the cooperative side of the mathematics teacher education programme. Apparently, up to now, ICT is often conceived as a powerful but individualistic tool, and it will be an important task of mathematics teacher education to introduce pre-service teachers in cooperative ICT-rich activities.

5. Studying Classroom Practice

The mathematics classroom and the school can be considered as a social setting in which norms are created and teachers and pupils try to ensure working practices and fulfil expectations. In the context of teacher education, the relationships between student teachers and their mentors as well as between student teachers and their peers are part of this environment and contribute critically to student teachers' learning (Jaworski & Gellert, 2003). When student teachers study classroom practice, this practice can be organised by experienced teachers, by themselves under the guidance of a mentor, or by other student teachers. A distinction needs to be made between the perspectives of someone who is teaching (or has just taught) mathematics and of an observer of this teaching practice. This distinction applies to situations in schools, where pre-service teachers watch the classroom processes initiated by experienced teachers or where teachers and teacher educators observe the pre-service teachers having a try in the classroom. From the perspective of the observer a detached analysis of what happens is possible, whereas the centred stance of practice requires a more intuitive grasping of situations in the classroom. Accordingly, the centred stance of teaching practitioners results, partly, in a shared generalisation of experience; outside observers, in contrast, argue on the grounds of symbolic objectives and general rules. These opposite perspectives, centred and de-centred, if not mediated tend to generate a reserved relationship that is not productive for learning from practice.

It is indeed important to overcome such opposite positioning. What is needed in order to be able to fruitfully analyse classroom practice in teacher education settings is a kind of re-centring. For that, it may first be necessary to disturb the natural grasping of classroom interaction to "perturb existing conceptions...of teaching and learning" (Mousley & Sullivan, 1997, p. 32) and to fathom the subtleties of the apparently straightforward course of action, which, for instance, is documented on videotapes or audiotapes. Second, analysis should be directed away from assessment and judgement of what can be seen on the tapes towards a contentious or consensual dispute of distinctions and effects. Disputes for convincing interpretations and for metaphors that guide future lesson design may develop a specific interpretative sense-making capacity. Briefly, a re-centring stance aims at semiotic self-regulation (see Raeithel, 1996) between pre-service teachers, mentors, and teacher educators.

These kinds of social positioning and the consequences for pre-service teachers' professional development are summarised in **Table 1.1.3.1**:

Table 1.1.3.1 Social positions with respect to classroom practice

Social positions	Perception	Shared knowledge	Developmental effect
The centred stance of teaching practitioners	Natural, intuitive grasping of situations in the classroom	Generalisation of experience	Consolidation and refinement of trusted and well-known skills, instruments, methods, etc.
The de-centred stance of observers	Detached analysis of familiar or unfamiliar actions	General rules, symbolic objectives	Description and assessment of classroom processes
The re-centring stance of semiotic self-regulation in a group that interprets teaching practice	Contentious or consensual dispute of the central distinctions and effects	Sense-making: disputing interpretations, using metaphors as guides	Modification or corroboration of approaches to teaching and learning and of action patterns

Different ways exist for overcoming the opposition of centred and de-centred stances. Sayac (2005) organises professional workshops, called "analysing professional practices", for pre-service primary mathematics teachers as a cooperation of primary schools and the Institut Universitaires de Formation des Maîtres (IUFM), in which pre-service teachers are prepared for their future teaching practice. The main aim of the workshop is to allow pre-service teachers "to get a better grasp of mathematics teaching at school, thanks to a reflexive analysis on their own practices" (Sayac, 2005). In order to reach a re-centring stance, Sayac has developed a scheme for the pre-service teachers' activities:

Pre-service teachers are dispatched in groups of 4 between classes of the same cycle (pupils aging between 3 and 6, or between 5 and 8, or between 8 and 11) in order to conduct lessons prepared at the IUFM on a specific theme. They will each in their turn play a different part so as to understand what is at stake during a mathematics lesson:

- One of the pre-service teachers acts as the class teacher and is responsible for the lesson, both in terms of preparation (choice of contents, organisation, management) and actual performance.
- Another pre-service teacher observes the lesson from the perspective of what is being learnt (relevance of the setting, of the didactic variables, of the instructions, of the organisation and duration of the lesson etc.).

- The third pre-service teacher observes the lesson from the pupils' perspective: How did the pupils react to the setting? Were they active or passive? Focused or unfocused? Did they face difficulties? Of what kind?
- The fourth pre-service teacher observes the lesson from the teacher's perspective: How did s/he manage the lesson at its various stages? Was s/he able to take all the pupils and all their reactions into consideration? What help did s/he bring to pupils facing difficulties? Which mediations did s/he use?

An audio recording of the session can be considered to show how the lesson went, how the different actors (teacher, pupils) interacted and what the atmosphere in the classroom was like. In order to facilitate such observations and to make them more fruitful, especially at the start of the year, I sometimes give the pre-service teachers an observation chart to fill in according to the various perspectives adopted.

During the evaluation sessions back at the IUFM, the pre-service teacher who had played the part of the teacher gives an account to all the other students of the lesson as he experienced it, and comments, if need be, on the discrepancies between what had been planned and what actually happened. The various perspectives adopted by the other pre-service teachers are then compared and contrasted under the guidance of the teacher educator who has obviously been attending the lesson (Sayac, 2005).

The evaluation sessions at the IUFM serve as the place for semiotic self-regulation between the four pre-service teachers who have directly experienced, although from different perspectives, the mathematics lesson under study, the teacher educator, and the other pre-service teachers who have been involved in the conduction and observation of other lessons. A re-centring stance is possible during these group meetings when the mediation between the pre-service teacher who has taught the lesson and the observers is successful.

Another attempt to reach a de-centring stance is made by Gellert and Krummheuer (2005). They are studying the ways shared knowledge is constructed among a group of experienced teachers, pre-service teachers, and themselves as teacher educators. They start from two basic theoretical positions:

1. Every practice of mathematics teaching and learning is a locally emerging process with open ends. The course and the results of this process depend on the students' and the teacher(s)' capacities to interpret and influence the interaction in their classroom. How students and teacher(s) understand each moment of the lesson is crucial for their scope and margin to shape a lesson's course. A mathematics lesson is exactly what those involved see in it. As a consequence, the following is suggested: if students and teachers were able to interpret the locally emerging processes of teaching and learning differently, then a different practice

of mathematics education would be possible. The broadening of teachers' interpretative resources is crucial for their professional development.
2. Interaction in everyday mathematics classes is a complex issue. Although interaction in the classroom is situated and its course is contingent upon the perception and realisation of those involved, the focus is on four dimensions that provide a structure for analyses of what happens in mathematics classes:

- Mathematical concepts, theorems, procedures, and models, which students and teachers talk about
- Arguments and argumentation patterns which students and teachers produce
- Patterns of interaction
- Forms of participation of active and silent students

According to Gellert and Krummheuer (2005), these dimensions facilitate differentiation between two opposite forms of interaction in the mathematics classroom, interactionally steady flow vs. thickened interaction. The first is characterised by fragmental argumentation, interaction patterns with inflexible role distribution, and less productive participation of all students; the second, in contrast, shows rather complete collectively produced arguments, flexible roles of students, and scope for their involvement in the educational process. These two forms provide different favourable opportunities for student learning. From Gellert and Krummheuer's perspective, teacher development may be seen as a path towards better opportunities for students' learning of mathematics, that is, to facilitate thick interactions that interrupt the interactionally steady flow of everyday mathematics lessons.

From this theoretical point of view, Gellert and Krummheuer organise a heterogeneous group consisting of pre-service teachers, practising teachers, and themselves as teacher educators:

> Based on these two assumptions we offered a 14-week mathematics education course, in which 5 teachers (from two primary schools, teaching 3rd and 4th grade mathematics) and 13 university students studying for a career as primary teacher took part. Participants were divided into stable subgroups of one teacher and two or three students each. The teacher and the two or three students met one day of the week in the school of the teacher. There, students observed the interaction between the teacher and the pupils and among pupils, videotaped parts of the lessons, prepared themselves (supported by the teacher) for teaching the class and taught the class (observed by the teacher).
>
> The whole group met one day of the week at university for what we call collaborative interpretation of classroom interaction. For each of these meetings, one subgroup selected about 15 minutes of videotaped (and transcribed) classroom interaction from the mathematics lessons in their school. The task of the whole group then was to reconstruct the interactional dimensions of the

> 15-minute scene. The goal was to analyse what happened in the episode, to find markers why things went as they went, and how the course of interaction could have developed differently—eventually with optimised learning opportunities for the pupils. The analysis aimed at uncovering the contingencies of the supposed natural and seemingly inevitable course of a lesson.
>
> Interpretation of videotaped classroom interaction is not a trivial task. If approached on the basis of common sense, videotaped scenes do not look radically unusual, and there seems nothing to be discovered under the surface. It is not before starting to scrutinise videotaped interaction systematically, that is to say using techniques for focussing on specific dimensions of the interaction, that one can see alternative paths through the possible ramifications of teacher(s)' and students' talk. For instance, some pupils' utterances that on the first view appear to show a lack of understanding of the mathematical problem to be tackled prove to be thoroughly rational, sense making and potentially helpful—they are just misplaced within the course of the arguments. In the first group meetings, we introduced three techniques for interpretation of classroom interaction: analysis of interaction, analysis of argumentation analysis, and analysis of pupils' participation (Gellert & Krummheuer, 2005).

A heterogeneous group interpreting classroom practice from a re-centring stance can be regarded as a promising approach for bridging the divide between formal knowledge (about, in this case, the contingency of classroom interaction and how to make use of it) and the practice of classroom teaching. The importance of interaction patterns and interaction mechanisms is likely to be overlooked from the perspective of concrete teaching in schools. Gellert and Krummheuer summarise that to analyse accounts of interaction is thus a crucial practice of learning from practice. The heterogeneity of the group seems to provide support for teachers' and primary teachers' learning from and for practice, although this heterogeneity still is a rather unknown quantity within research on mathematics teacher education.

In contrast to the two first examples discussed in this chapter, the last example is related to the education of future secondary mathematics teachers. It is more strongly focused on a teaching technique: the teachers' generation of questions. It offers insight in reflections about the issue of what teacher educators might do "to guide pre-service teachers toward their own thinking about questioning for mathematics understanding instead of pre-service teachers searching for authenticated knowledge in this matter" (Rosu & Arvold, 2005):

> The study of questioning was initiated at the beginning of the year when pre-service secondary mathematics teachers questioned their abilities to imagine and practice successful questioning for understanding in real mathematics classrooms. As a response to these concerns, we proposed the study of

questioning for the next teaching experience. The study of questioning addressed the content of the experiences in teacher education much more than prescriptions on structure and procedures of the study of questioning.

In the first stage, pre-service teachers were paired and each pair prepared a study of a specific issue in questioning. Special classes were set aside to discuss the focus questions and the design of studies. Readings enriched the discussions. Novices focused both on students' and teachers' questions in the field, and on their own practices.

For two months, we monitored and supervised pre-service teachers' studies. Electronic discussions within a learning community of pre-service teachers, and cooperating and mentor teachers prompted observations and debates in the study of questioning.

In the second stage, during their second semester student-teaching experience, each pre-service teacher, based on her/his experience with the previous semester investigations, designed a more focused study of questioning in practice. Weekly class meetings and discussions accompanied these studies (Rosu & Arvold, 2005).

All three examples have demonstrated that intensive and intelligent studies of classroom practice value theory and practice not as distant poles but as reflexively connected elements of knowledgeable activity. As Sayac (2005) concludes, "Educating teachers through practice and for practice should therefore take pride of place in the initial education of teachers because teachers can thus be initiated to the analysis of their own practice, using concepts developed by research in mathematics education, and this will facilitate the learning process of their future pupils". The study of classroom practice is very different from any unreflective field-based experience.

6. Limitation

A single course experience cannot, of course, create comprehensive or permanent changes in teachers' perceptions of mathematics and mathematics teaching nor will such a singular experience significantly affect teachers' classroom practice. Sullivan (1989), for instance, demonstrates how teaching newcomers, who have successfully worked with interesting activities along current approaches to mathematics education during their initial teacher education programmes and who have shown high levels of self-reflection, fall back on "traditional" methods of mathematics instruction shortly after having been employed. Apparently, schools seem to be very effective in integrating new teachers in the prevailing culture of the school.

However, expanding the realm of possibilities does not simply aim at fruitful yet isolated course experience. The ultimate purpose of presenting interesting examples from teacher education practice is the construction of a more reflective mathematics

teacher education culture, within which teacher educators and future teachers can engage in developing and elaborating mathematically and socially sound conceptions of mathematics teaching and learning. As a matter of fact, this is not a short-term project.

A last, and critical, remark: the examples of mathematics teacher education practices presented and discussed at the study conference may, on the one hand, be regarded as the status quo of innovative developments in teacher education. On the other hand, there seems to be reason to assume that these examples, although diverse in nature, reflect a kind of occidental mainstream in mathematics teacher education research. The diversity of mathematics teacher education practices and of underlying intentions of these practices is, of course, much broader. For instance, in some places there is an ongoing discussion whether and how to include ethnomathematical practices and reflections within teacher education programmes. In other places, particularly in countries of social transition and/or political transformation (at the time of writing, e.g., Venezuela, Bolivia), the request for a critical mathematics teacher education is considerably high. It is, perhaps, a negative side effect of the high quality and the institutional character of an ICMI study conference that not all developments and discussions, particularly from the periphery, can be included in conferences and official reports.

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Chapter 1.1.4
Learning to Teach Mathematics: Expanding the Role of Practicum as an Integrated Part of a Teacher Education Programme

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1. Introduction

Teacher education programmes at tertiary educational institutions traditionally comprise three key strands—disciplinary studies, educational studies, and teaching practice (Comiti & Ball, 1996). The aim of these strands is to develop an integrated competence in student teachers and is often referred to as teacher knowledge. Winsløw and Durrand-Guerrier (2007) name the respective target knowledge components as content knowledge, pedagogical knowledge, and didactical knowledge, noting that each component "may occur with different emphases on theory and practice" (p. 7) and are viewed, in terms of weight and organisation, differently within different cultural traditions. In teaching practice, as an activity within a teacher education programme, all these components come into play in the very contextual setting where they are supposed to be functional. This is where student teachers can experience a test of the viability of the level of their own teacher knowledge. It is often witnessed by student teachers that during teaching practice, working along with an experienced practising teacher, is when you really learn something about teaching (Bergsten & Grevholm, 2004; see also Johnsen Høines & Lode, 2007); another quality is added compared to the theoretical courses on theories of education or teaching methods. The relevance of teaching practice, especially when student teachers are given the opportunity to pilot new didactic proposals they have contributed to develop, has been shown to be very high in different national teacher education contexts, even when the differences are significant in terms of structure, organization, and curriculum (Favilli, 2006).

The term "teacher training" reflects an apprenticeship paradigm for the development of teacher knowledge. Historically, for primary education the main part of

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the preparation of teachers has also consisted of teaching practice. The apprenticeship model has been criticized for supporting a continuation of existing practices (Lanier & Little, 1986; see also Mewborn & Johnson, 2005). With a modern view of a scientifically based higher education a wider spectrum of academic courses makes up, along with teaching practice, what is now normally called teacher education rather than teacher training (this shift in terminology is also discussed in Bednarz & Proulx, 2005). The change of discourse is also reflected in a change in the view of the role of teaching practice and, as a consequence, its organisation within teacher education programmes.

When talking about teaching practice within an institutionalised teacher education programme, we will use the term "practicum", defined by Wikipedia as "a college course, often in a specialized field of study, that is designed to give students supervised practical application of a previously studied theory". This definition emphasizes the connection of practice to theory, excluding a "blind" practice for its own sake, but does not give full credit to the "silent" knowledge of the community of practicing teachers earned by experience of teaching, and reflections on this experience, from which the student teacher can profit. The definition does not exclude the use of practicum as an empirical field of study for the student teacher, making it possible to make observations and data collection in relation to tasks provided in theoretical academic courses, thus also providing feedback from practice to theory. Thus, in addition to the general definition given previously, by practicum we mean the work of a student, enrolled in a teacher education programme, as a practising teacher. This work takes place in a school under the supervision of an experienced mathematics teacher, the mentor. The work is organised as a result of cooperation between an institution that offers teacher education and a school. The mentor who supports the student has been given this task as a formal part of her or his work in the school. There is an explicit agreed aim with the practicum, which may also include assigned tasks of an investigative character.

As mentioned previously, teacher education normally has as one part of the programme a practicum. Historically the education of teachers has been organised in many different ways depending on the intended school level and educational traditions as well as societal and cultural constraints (see, for example, Bergsten & Grevholm, 2004; Winsløw & Durrand-Guerrier, 2007). In the papers presented at the 15th ICMI Study we did not find much about how to organise the practicum, although we are aware of the fact that the collaboration between schools and teachers on one side and the teacher education institution on the other is not at all unproblematic but demands careful work if it is going to function well for the students. Teachers in schools are in a situation requiring hard work and are often not so eager to take on another burden, such as being a mentor for a student teacher. In addition, the extra work is often not paid by the school, although the institution has to pay the school for the collaboration. Normally this part of teacher education is the most expensive part, as it is a resource for the individual and not a whole group. This financial commitment points to the fact that a practicum is considered necessary in teacher education. What aspects, then, of the teacher profession can one not be expected to learn only through theoretical studies?

As pre-service teachers often revert to "teaching styles similar to those their own teachers used" (Brown, Cooney, & Jones, 1990, p. 649), Lerman (2001, p. 48) notes that "courses do not provoke students to confront their naive notions of teaching mathematics" and sets out the student teacher's development of an identity as a mathematics teacher as the critical issue. Such development may be supported by critical reflections on experiences from practicum of existing practices in schools and from promoted practices displayed at the teacher education institution. However, the student teacher must then co-exist within different discourses that are sometimes conflicting and value laden as well as subject to social power issues, which also influence "modes of operating, knowledge, and positionings" in the school practice context (Walshaw, 2004, p. 68).

At a very concrete level, what student teachers sometimes worry about in the beginning of their studies is how they will be able to work with pupils. They cannot easily imagine what to expect from pupils concerning earlier knowledge in mathematics, their thinking and reasoning, and their behaviour in a mathematics class. It is therefore often a relief for them when they find out, during the first practicum, that they are able to communicate with pupils and explain to them in ways that are well received (Bergsten & Grevholm, 2004).

One key aspect of a teacher's work is the practice of classroom management. In this we include the student teacher's ability to establish communication with the pupils individually and as a group, to talk to pupils about the mathematics content, to listen to pupils' ideas and reasoning and try to understand it on the spot, to be able to respond in a way that the pupils find helpful and meaningful, and to use a professional language that is functional in the communication with pupils (Grevholm & Bergsten, 2005). The ability to listen, to hear pupils, is rarely dealt with explicitly in teacher education, although it is highly important and often critical as informal assessment of pupils' knowledge during normal class work (Wallach & Even, 2005). The student teachers' beliefs about mathematical knowledge and teaching, as well as attitudes towards the pupils, come into play here, thus providing a basis for further reflection and elaboration in courses linked to the practicum. All these aspects of classroom management are difficult to learn how to handle by theoretical studies only and thus constitute some of the obvious aims for the practicum.

Another aspect that teacher educators would investigate when visiting a student teacher during practicum in school would be if the lesson resulted in learning. It is possible to organise a class that seems to be running well and smoothly and where pupils seem to be pleased but where little learning actually takes place (Goodchild, 2007). The question is if the student teacher is able to care for the important, relevant issues in class and not only for superficial aspects that do not really matter when it comes to pupils' mathematical learning.

One critical issue is the weight given to the practicum in teacher education programmes in terms of volume as well as how it is being evaluated or assessed. As an example, what kinds of criteria are used to evaluate the student teacher's work during practicum, and who is doing this evaluation? What is lacking in a student's abilities and competencies when not given a pass in the practicum, and how does this relate to when during the education the practicum takes place? As these issues

highlight what aspects of teacher knowledge are valued by the different programmes, they deserve serious attention in discussions about mathematics teacher education.

In the following sections some interesting examples presented at the study conference of ways of working with and through practicum will be discussed. Some of these concern how the educational and motivational payoff of practicum can be increased by the choice of structure of the programme. In addition, theoretical tools related to practicum activities have been developed and integrated in the education. Examples of more specific practices are also presented, and some questions are raised concerning issues of cultural differences in relation to practicum. Finally, some conclusions are drawn pointing to the expanding role of practicum as an integrated part of a mathematics teacher education programme.

2. Structural Ways of Using Practicum

The term "structure" in connection to a teacher education programme can apply to several dimensions, such as the balance of different components of the programme and their order and integration in terms of courses and practicum, combinations of teaching subjects, and target age levels of students to teach, as well as more fine-grained structural elements of formats of tutoring in lectures, classes, or small activity groups. Another dimension refers to who is teaching pre-service student teachers—what is the role of mathematicians, general educators, mathematics educators (didacticians), experienced teachers (mentors), or others? Yet another dimension is the issue of research—what is the balance of research-based and experience-based aspects of the education, including the role and character of the diploma thesis and teaching practice (practicum)?

Due to organisational and institutional factors, including traditions and values, a complex professional education such as that of future teachers is likely to result in compartmentalised knowledge, where for example mathematical content knowledge, didactical knowledge, and experiences that form practicum have no or only weak connections. Such a compartmentalisation of teacher knowledge, which may exist both on the organisational level and as personal knowledge within the student, has been termed a didactic divide (Bergsten & Grevholm, 2004). A programme to develop a unified organisation of an educational knowledge in mathematics needs to merge the divide between content and didactical knowledge, as defined previously (Bergsten & Grevholm, 2005). One component of a teacher education programme with a potential to bridge this didactic divide is practicum, including not only its professional content, but also its structural organisation. One key issue is then the integration of practicum with other course components; another is the kind and level of responsibility given to the student teachers.

The notion of practice is in the centre of the teacher education programme described and discussed by Bednarz and Proulx (2005), based on a principle of "learning-in-action rather than learning-about-action" (2005, p. 2). By a planned integration of practicum with other course components, a pool of observations and

experiences of teachers' and student teachers' enacted knowledge in classroom settings make possible an emphasis on actual educational situations. Such integration is further supported by a practice in which mathematics educators teach both the mathematics-content courses and the didactics courses and take part in the supervision of the student teachers during practicum. Using a principle of contextualized knowledge construing, teaching situations are placed in a cycle of "planning based on a conceptual analysis of a mathematical notion, a priori analysis of curricula and usual teaching approaches, the construction of a repertoire of chosen problems, and a classroom experimentation" (2005, p. 2). By adding tasks of reflective analyses and collective discussions, with such sequences recurring during the whole programme, the student teachers are supported to develop a personal conceptual reference framework. The functioning of this programme has been investigated in a case study by Proulx (2003), where it was observed how the five participating student teachers interpreted the programme very differently although in a coherent way. It was viewed as a source of potential teaching resources, an authoritative acceptance of principles and content, recognizing and supporting one's own implicitly used principles, as a philosophy of teaching using general principles rather than specifics, or as a model-in-action. These differences may be due to the student teachers' different backgrounds and visions, and make it necessary to "move away from an intention of control" on what kind of "good practices" to promote in a mathematics teacher education.

An important issue for student teachers is their experience, already during practicum, of contributing to the growth of their pupils' mathematical knowledge and self-esteem. To account for this, Tirosh and Tsamir (2005) describe how the evolution of their teacher education programme designed a third-year practicum at a school with a low socioeconomic population, where the programme took full responsibility for the geometry teaching in grade nine. The student teachers working in pairs in small-sized classes, supported by an experienced mentor, were highly motivated by this arrangement and put forth strong efforts in their teaching.

To support the development of the student teacher's identity as a mathematics teacher, an elaborated integration of practicum with the other parts of the programme may be efficient but not sufficient if there is not also a personal commitment and an experience of having reached out to the pupils and seeing a result of this commitment. The content and structure of practicum in a programme designed to develop a personal conceptual reference framework, as described previously, therefore seems critical.

3. Theoretical Tools to Use Practicum

One way to expand the role of practicum in teacher education is to integrate research-based theoretical tools with activities in the programme. Such tools may help the student teachers analyse and reflect upon their practicum experiences in

a more focused and systematic way and thus deepen the understanding of critical aspects of the teacher's role in the mathematics classroom. Important here is the work of the mentor as well as the student teacher's post-teaching discussions with the mentor and the teacher educator, which take place within many programmes. Since such conversations traditionally tend to have an evaluative character in terms of normative statements, rather than focus on epistemological aspects of mathematical knowledge and learning, they risk hindering the student's development of teacher identity. To avoid this, Johnsen Høines and Lode (2007) investigated didactical conditions for a subject-based discussion to support a more reflective approach. To learn, in the sense of developing a teacher identity, from imitation of the mentor as a model teacher, the student teacher also needs to understand the rationale behind the activities of the mentor (Nilssen, 2003).

A student teacher, when enrolled in a formal teacher education programme, is typically participating in at least three socially organised practices, linked to the corresponding mathematics teaching knowledge components: content knowledge; pedagogical knowledge; and didactical knowledge, as discussed above. Due to institutional traditions and different epistemological emphases, these practices have developed their own specific discourses, which may be experienced as contradictory by the student teacher, as well as contribute to the development of their professional identities (Lerman, 2001). To analyse practicum experiences, and the relation between the pedagogical models offered at school and those "taught" at the academy, Goos (2005a) suggests a theoretical framework based on Valsiner (1997), integrating Vygotsky's conception of a zone of proximal development (ZPD) with a zone of free movement (ZFM) and a zone of promoted action (ZPA). Here, the ZFM for a student teacher during practicum accounts for the environmental constraints on action and thought, such as the characteristics of his or her students, curriculum requirements, and teaching resources. What the teacher educator at the academy, as well as the mentor and other experienced teacher colleagues, promotes as desired teaching approaches make up the ZPA. For the student teacher to develop a teacher identity, it "is important that the ZPA be within the novice teacher's ZFM, and is also consistent with their ZPD" (Goos, 2005a, p. 2). As a complicating factor, this development may be influenced, during practicum, by conflicting ZPAs, as represented by the teacher education programme and the mentor at school. The strength of this framework to help students to analyse their practicum experiences and relate them to coursework at the academy, is highlighted by Goos (2005a,b) by an example from a research study in an Australian context.

As an alternative to the prevailing apprenticeship model for teacher education, Mewborn and Johnson (2005) argue for the use of the notion-assisted performance (Feiman-Nemser, 2001) to engage student teachers in central tasks for their pre-service education. In line with the Vygotskyan conception of ZPD, such assisted performance provides opportunities to enable them to "learn with help what they are not ready to do on their own" (ibid., p. 1016) rather than a mere practice of what they will do as in-service teachers. This may prevent the commonly observed alignment to own experiences after the apprenticeship period, that is, to "teach as you were taught". Examples of assisted-performance tasks provided by Mewborn and

Johnson include "reading and discussing an article,... working one-on-one with a child for eight weeks, and observing an experienced teacher" (2005, p. 2).

Recognising that post-observation meetings during practicum between student teacher, mentor, and teacher educator often tend to focus on classroom management rather than on aspects of how mathematical knowledge per se has been handled during the lesson (Brown, McNamara, Hanley, & Jones, 1999), Rowland, Thwaites, and Huckstep (2005a) suggest an empirically based framework called the "Knowledge Quartet", aimed at giving structure to such discussions of teachers' mathematical knowledge in the classroom. The first dimension of foundation refers to subject-matter knowledge as well as beliefs and understandings related to the teaching and learning of mathematics developed during academic coursework. During lesson planning and actual teaching, teachers' "knowledge-in-action" defines the second dimension, transformation. Of interest here is, for example, how examples are chosen and used to support student learning. How the teacher provides links and handles different cognitive demands of separate parts of mathematical content constitutes the third dimension of the quartet, connection. Finally, to account for the unexpected, for decisions impossible to plan for about developments of classroom activity, the dimension of contingency completes the quartet. Elements of mathematical knowledge in lesson episodes can be captured and understood in discussions at post-observation meetings during practicum, when structured by the four dimensions of the Knowledge Quartet (Rowland, Thwaites, & Huckstep, 2005b).

All these examples highlight the strength of using different theoretical tools for designing and framing activities with a potential to integrate formal courses and practicum to a functional basis for developing teacher knowledge and identity.

4. Specific Ways of Using Practicum to Develop Teacher Knowledge

Within different organisations of practicum and theoretical frameworks for analysing teaching practice, more specific activities have also been developed which may contribute to the expansion of the role of practicum. Examples reported here deal with gaps between planned and actual classroom activities, the use of stories of practice, questioning as a tool in teaching practice, and establishing communities of interpretation of classroom interaction.

Recognising that a common experience in practice is gaps between the (student) teacher's planned and actual activity in classroom episodes, DeBlois and Maheux (2005) focus on how student teachers during practicum explain and what they learn from such gaps. A discussion team, comprised by the student teacher, the mentor, the school's special education teacher, and a researcher, met regularly before and after the testing of planned activities. The meetings were structured by phases of narration, analysis, and synthesis. From these meetings it could be noted that the student teachers used four types of adaptations to handle such gaps. Using a projective adaptation, the student teacher grabbed an utterance or a question from

a student to put questions or pursue further discussion. When the students were expected to manage difficulties by themselves, a withdrawal adaptation was sometimes practiced, observing students doing their mistakes. By prompting students to adjust to specific ways of proceeding, a normative adaptation to an experienced gap was used. Lowering expectations on students, simplifying tasks, or not requiring explanations are examples of avoidance adaptations. The team-meetings format also triggered discussions about what factors cause these different ways of gap adaptation, and the student teachers were "able to recognize the devolution of the teaching situation... the 'taking charge' of classroom activities, and the student teacher's projection into his/her professional practice" (DeBlois & Maheux, 2005, p. 5).

As a means for analysing practicum and provide one's own lived experiences as cases for reflection in theoretical didactic courses, Chapman (2005) suggests an approach of using stories of practice in pre-service teacher education. Rather than judgements about good or bad teaching, the focus is on sense making. As the first stage of a sequence of four, student teachers are asked to write one story each of "good", "bad", and "memorable" teaching from their own teaching during practicum or from their own observations of teaching. The story is to include a complete mathematics lesson with as much detail as possible, including what teachers and pupils have said, but should be descriptive rather than normative. Details not remembered are to be filled in with what makes sense, not to leave gaps. The second stage is one of initial self-reflection, where the student teachers write journals on why they think their stories represent good or bad teaching, which they share and discuss with their peers, providing readings for what stories they like or don't like. During the third stage the stories are used during the semester to interpret theory and for the analysis of actual practice during practicum with a focus on making sense of mathematical content and discourse in the classroom as well as alternative approaches. The fourth and final stage, at the end of the course/semester, aims at a final self-reflection by rewriting the previous story "in the way he/she would want it to unfold", in order to "provide an alternative way of conducting it in term of engaging students in the content to facilitate deep understanding of it" (Chapman, 2005, p. 4). The student teachers then write journals to compare their two stories, to share and discuss with their peers. Data from an observed sequence with 26 student teachers showed that this approach of writing stories of practice provided a constructive means to articulate their thinking of mathematical teaching and learning in a holistic way, with self-reflections prompting conflicting beliefs and shifts of thinking. The analysis of practicum initiated an increased awareness of critical aspects of teaching not previously noted, leading to a more inquiry-oriented approach and recognition of the need of a deep understanding of mathematics to be able to support their students' learning.

Since questioning is one key tool in teachers' practice to promote and reveal student learning, its place in teacher education needs illumination. Rosu and Arvold (2005) report on a study of questioning that took place in a secondary mathematics teacher education programme. After an initial sequence of investigations into questioning, the student teachers studied questioning in practice during their second semester practicum, including meetings and discussions. Whether the focus

in these discussions were on learning for or in practice, these studies "generated a milieu of learning appropriate for the multiple meanings and contexts of teaching experiences" (ibid., p. 4). It was observed that an inquiry approach was supported by the study of questioning, which helped student teachers develop knowledge on students' mathematical understanding. However, this focus on questioning also created a milieu in teacher education where questioning as practice and questioning as theory do not come into conflict. To develop and maintain an inquiry stance in teachers, the study reported supports a practice of questioning as a learning milieu.

Based on the two assumptions that a lesson in mathematics is "exactly what those involved see in it" and that classroom interaction is very complex, depending on the mathematical content under discussion, lines of arguments used, interaction patterns, and how students participate, Gellert and Krummheuer argue that a focus on a "collaborative interpretation of classroom interaction" (2005, p. 2) may be productive for learning from teaching practice. To be able to "uncover" what was behind the development, or flow, of a lesson, a group of teachers and student teachers, along with the researchers, met to analyse a chosen videotape of a fifteen-minute lesson sequence. To give structure to the interpretations produced, three techniques developed in mathematics education research were adopted, that is, interaction analysis, argumentation analysis, and participation analysis. Seen as members of a "community of interpretation" (2005, p. 3), this group also involved in different communities of practice moved during the meetings from peripheral to full participation in this community as they become more competent in classroom-interaction interpretation. By using a heterogeneous community of interpretation it was possible to make different interpretations of classroom interaction visible and as a consequence open up for change and development of teaching approaches. The rationale behind this outcome relates to differences and changes of perspective, contrasting the "centred stance of teaching practitioners" and "de-centred stance of observers" in the "re-centring stance of legitimate self-regulation of a community of interpretation" (2005, p. 5).

The didactical power of practicum rests on experiencing oneself as an agent inside the classroom taking part in and affecting its flow. By reflecting jointly, within this context, on one's own adaptations, stories, questioning, and interpretation of classroom interaction, an increased awareness of the complex processes of teaching and learning can emerge and contribute to shaping one's identity as a mathematics teacher.

5. Practicum and Issues of National and Cultural Differences

National differences between teacher education programmes in terms of structure, organization, and curriculum represent a crucial issue when speaking about teacher education in general and practicum in particular. Several questions should be answered to better understand mathematics educators' and student teachers' habits, behaviours, and attitudes as far as the practicum is concerned. Is the practicum

part, for example, of a university degree programme in mathematics in a faculty of sciences or in mathematics education in a faculty of education? Or is it part of a post-graduate university programme? What is the volume of practicum in terms of time, and is it organised in longer or shorter periods? What kinds of activities are required by the student teacher during practicum, in terms of observations, own teaching in class, specific tasks such as assessing pupils' mathematical knowledge or collecting other empirical data, or reporting from practicum? During their practicum, are student teachers provided with supervisors by the institution organising the teacher education and mentors by the school? How do supervisors and mentors interact in particular and interrelate with the whole educating team in general? In several papers submitted to this ICMI study it is difficult to find explicit answers to even just a few of these questions.

Another crucial question could refer to the history of teacher education programmes at the national or local levels, as mentioned previously. These programmes hold a (sometimes very) long tradition in some countries, whereas they have been (sometimes very) recently introduced in others. These differences set different constraints to didactical developments and reflect and affect the local school, educational, and, even, societal contexts, with their cultural values and should be kept in mind because they are present in the schools and the classrooms, where the practicum takes place (e.g., Skott, 2005). Some examples from the study conference will illustrate this.

Considering the previous remarks, it is not surprising to read the following realistic and honest comment:

> Schools' attitudes towards practicum have not been pleasing recently. Cooperating teachers show indifference towards student teachers, and school administration has deemed student practicing as disrupting school schedule. Part of this problem emanates from students' unbecoming behavior. Some student teachers do not take their practicum seriously, resulting in inadequate preparations of lesson plans and scheme of work. Moreover, timetable clashes result in inadequate supervision (Garegae & Chakalisa, 2005).

Here the authors refer to Botswana. Is it a single case? Where else can similar situations be found, and why?

Another issue concerns the use of information and communication technology (ICT) in mathematics teacher education: How common is it, and how is it an integrated part of the practicum? In Kadijevich, Haapasalo, and Hvorecky (2005) this strong "official" complaint can be found: "This important issue of preparing teachers [to the ICT] has not been recognized by the ICMI Study 15. . .whose Discussion Document, to the authors' surprise, doesn't even use the words 'computer' and 'technology' ". Nevertheless, the topic is raised in several other papers (e.g., Bairral & Zanette, 2005; Miller, 2005), recognizing its power of shaping the structure of the educational environment.

Other critical issues relate to interdisciplinarity and diversity, as expressed in Carneiro Abrahão and de Carvalho Correa de Oliveira (2005). Philosophical premises of the courses as well as of the discipline of mathematics "are related to the interdisciplinary practices and the development of opportunities to consider diversity in classes" (2005, p. 1). Centred around research activities in a real context

the education is aiming to learning to select "priority objectives, and appropriating resources and strategies in teaching mathematics in an interdisciplinary approach" (2005, p. 3), and the knowledge increase "includes an evolution in the connection among the disciplines" (2005, p. 5). In this paper, interdisciplinarity is thus strongly emphasized and rooted in a pedagogic philosophical idea. What do the different teacher education programmes in mathematics provide to the student teachers under this aspect? Which obstacles, if any, are faced in the implementation of an interdisciplinary education, namely, when practicum is concerned? How can they be removed? Is it just a matter of tradition or of educational culture? For instance, in Italy, despite lower secondary-school mathematics teachers having to teach experimental science as well, the two subjects tend to be rigidly separated in the pre-service training programmes (Favilli, 2006) and, as a consequence, in the class practices. However, as reported by Novotná and Hofmannová (2005) for the Czech Republic, mathematics and language student teachers are being trained to cooperate in view of teaching mathematics through the medium of the English language, using the Content and Language Integrated Learning approach.

One aspect of diversity in classrooms concerns intercultural education in mathematics, raised in Favilli (2005): "Policy makers and educators are increasingly concerned with the inclusion of minority culture pupils in the classrooms. Mathematics teachers have started to consider the ways of dealing with multiculturalism in the class. In several countries, such as Italy, they complain about the lack of refresher courses and didactic materials." How are initial teacher education programmes in mathematics dealing with this issue? As the classroom is the place where the actual cultural commonalities and differences can be seen, the practicum is even more crucial for this kind of training. In a multi-cultural classroom the passage from the theory (even when supported by a good introduction to the intercultural education issues) to the practice can be even harder than in an "ordinary" classroom.

6. Conclusions

The examples presented here reflect only some of the ongoing developments of practicum in mathematics teacher education taking place in different parts of the world. However, they all reflect a move away from a teacher training paradigm of teaching pre-defined teaching skills and a fixed body of content knowledge towards a teacher education paradigm of developing educational knowledge in mathematics. This accounts for a shift both in the target knowledge of mathematics teacher education and how to achieve it. Within this shift, the main features of the picture that emerges of the role and practice of the practicum are both its complexity and its educational potentials. The latter explain why teacher education programmes continue to offer a practicum although it is expensive and difficult to organise in ways that fulfil the demands from students and the institution.

It is obvious that there is a general agreement that some parts of the teacher profession are better learned in a classroom. There is also the question of how to

balance theoretical studies and time spent in the classroom. This has varied over time and also between teacher education programmes in different countries. Student teachers often claim that this part of their education is the most important, and the study conference papers discussed here explicate the rationales hiding behind these beliefs. Other professions also include practice, such as nurses, medical doctors, lawyers, priests, and police officers. What kinds of issues concerning practice are relevant for all these professions and their practicum? What can mathematics teacher education learn from research about practicum in these other professions as well as from the education of teachers in other disciplines?

Issues of differences in cultures and school systems between countries, which may set critical constraints or open up possibilities with regard to the structure and kinds of activities that are viable in a teacher education programme, have only been raised briefly in the discussion here but with an emphasis on their importance. The focus has rather been on the kind of teacher knowledge needed for teaching mathematics in schools today and how to support the development of such knowledge in student teachers within an institutionalised educational programme. A critical issue concerns how to organise an education that supports the development of a teacher identity. This sets demands on the student teacher to navigate between constraints, free movements, and promoted action, for which practicum is the key arena. An elaborated integration of practicum to other parts of the teacher education programme is needed to base this development in a personal conceptual-reference framework. Practice-based reflecting conversations with participants in their education need a theoretical basis and a focus on the diffusion of mathematical knowledge to contribute to personal and viable teacher knowledge.

The contributions at the study conference presented in this chapter all witness how the role of practicum can be expanded as an integrated part of a mathematics teacher education programme, as well as the need of this educational development to accomplish the aims of the complex enterprise of preparing students for the teaching profession. Structure, formats, and activities that merge didactic divides between different components of the education seem to be at the core of these examples, with a common aim to foster the development of a personally based teacher identity in future teachers of mathematics.

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Theme 1.2
Student Teachers' Experiences and Early Years of Teaching

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The preparation of teachers of mathematics takes various forms across the world, but there is no doubting its importance in every country. There were many papers addressing aspects of initial teacher education in use across the world presented at the 15th ICMI Study. In this section we have provided an overview of research perspectives to represent the important contributions of the participants.

In Chapter 1.2.1 Stephen Lerman looks at research studying student teachers' voices and their beliefs and attitudes. The field of teachers' beliefs and attitudes is both well known and substantial and has been studied in different ways for more than two decades. The research presented at the conference contained many new perspectives and in particular focused on interpreting students' views and ideas. Lerman draws on methodological issues to structure the chapter: interpreting students' voices; relations between attitudes and beliefs "in theory" and practice; systematic observations of classrooms; and challenges for teacher education.

In Chapter 1.2.2 Merrilyn Goos looks at school experience during pre-service teacher education from students' perspectives. In particular she examines notions of identity and the development of communities of practice as contexts of interpretation to make sense of students' perspectives in a manner that informs research and teacher education.

In Chapter 1.2.3 Carl Winsløw examines students' first years of teaching. Viewing this period as a key transition in epistemological, institutional, and personal levels, once again in the different systems across the world that were represented by participants at the meeting. He examines, in particular, the ways in which ideas that students encounter in their teacher education programmes might sink or swim in the reality of schooling and in the face of the often-encountered inertia of schools and established teachers.

Chapter 1.2.1
Studying Student Teachers' Voices and Their Beliefs and Attitudes

Stephen Lerman, *London South Bank University, London, England, UK*,
and S. A. Amato, N. Bednarz, M. M. M. S. David, V. Durand-Guerrier, G. Gadanis, P. Huckstep, P. C. Moreira, F. Morselli, N. Movshovitz-Hadar, I. Namukasa, J. Proulx, T. Rowland, A. Thwaites, C.Winsløw

1. Introduction

Learning to teach mathematics is a complex undertaking, and in the last twenty years there has been a great deal of research looking at aspects of the process. There are many ways one might structure an analysis of research on learning to teach mathematics. It is clear, though, from all the research on learners in all kinds of situations that what student teachers bring to their teacher education courses in terms of prior knowledge, experience, attitudes, beliefs, goals, fears, hopes, and expectations has to be a key factor in preparing for and teaching those courses and hence for research. This particular focus for research in our field is not new; my own doctoral studies, completed in 1986, looks at connections between student teachers' beliefs about the nature of mathematics and their perceptions of teaching mathematics (Lerman, 1990).[1] It remains, however, of great importance, and there are new insights drawing on a range of theoretical frameworks emerging in the field.

Central to research in the study of student teachers' attitudes and beliefs and any changes in those beliefs during pre-service teacher education courses are issues of methodology. Access to student teachers' beliefs and experiences is inevitably through their voices, expressed in interviews, conversations, and their writing, but the interaction of beliefs and practice is a necessary consideration too. First, regarding students' voices, what must concern us is how to read across a number of stories in order to be able to say something about how these voices are produced (Arnot & Reay, 2004). Without that focus we produce a spiral of more and more detailed stories with no possibility of making sense of the data (see Brown & McNamara, 2005, for an excellent example of the struggle for an appropriate theoretical framework for analysing student teachers' voices). Second, there is always a gap between what people say about what they do and what they actually do in their practice (Lerman, 2002). Research needs, therefore, to be aware of that gap

[1] All references are to papers presented at the ICMI 15 study conference unless otherwise indicated by date. A list of papers appears in the reference section of the chapter.

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and find ways to bring descriptions of practice as close to that practice as possible. Video-recording lessons and interviewing the (student) teacher using those videos for stimulated recall immediately after is one way that has been used (Clarke, 2001). Third, choosing to observe teachers' practice as a way of bringing together these two domains draws the researcher into interpretation of the observations, calling for some form of systematic procedure with explicit rules for recognition and realisation (Brown & Dowling, 1998). These are not insurmountable difficulties of course, as can be seen in the research presented at the 15th ICMI Study being reported upon in this chapter, for example. These methodological issues remain, however, as considerations when planning, carrying out, and reviewing research on student teachers' attitudes, beliefs, and practice.

As a final comment in this introduction, I want to note the challenge for teacher educators regarding the theme of this chapter. Clearly, one's task is to help the student teachers become the best mathematics teachers they can be, however one might define that—opening up a space for the play of ideology of course. Whatever position one takes, however, attitudes, beliefs, and dispositions, affected or determined by prior experiences, are likely to be obstacles or affordances to a student's achievement of that goal. This gives rise to talk of "changing student teachers' beliefs" and other, potentially coercive, objectives. At the same time, the teacher educator has a wealth of experience, informed by research, and should not shy away from drawing on that experience. Furthermore, there are likely to be regulatory constraints on what is perceived by government to be good mathematics teaching, and there may also be a wealth of cultural knowledge and practice to be acquired by students. One might best see these contradictory elements as a tension which needs to be constantly engaged, and researched, in the practice of mathematics teacher education.

In what follows, I will examine each of these methodological issues in turn, drawing on the papers presented at the 15th ICMI Study, namely, interpreting students' voices, relations between attitudes and beliefs "in theory" and practice, systematic observations of classrooms, and challenges for teacher education.

2. Interpreting Students' Voices

As suggested previously, eliciting students' voices in a manner that informs research and teacher education requires an approach that draws across cases to reveal how the positions student teachers narrate are produced and gives indications of which actions might be relevant for the teacher educator. Questionnaires and interviews are the common source of students' voices, but there are other approaches. For example, "two of the assignments (one at the beginning and one at the end of the course) asked students to describe their view of mathematics and to discuss an appropriate metaphor for their personal experience with mathematics" (Gadanis & Namukasa, 2005). Another approach is found in engaging students in solving mathematics problems. The researchers might then observe students at work, since "the analysis of answers (to a questionnaire or an interview) can give information on

teachers' explicit conception of mathematics, while the observation of their practice can give an insight into their implicit...conceptions" (Morselli, 2005). Alternatively, students can engage in mathematical activities, reflecting subsequently on implications of their experiences for school mathematics.

In a comparison between French and Danish teacher education students Winsløw and Durand-Guerrier (2005) comment:

> It's quite surprising that all five French pairs insist on the importance of finding a "real-life" situation to explain the sign rule (for $(-2).(-3) = 6$); given their mathematical formation, one could expect them to... search more for a mathematical explanation of the non-arbitrary character of this convention.
>
> The Danish pairs have encountered this problem in their own formation, and the two pairs who explicitly realise that their "explanations" are false or just mnemonic rules, seem to conclude that the problem is somehow too difficult for them and hence for their pupils. As one of these four students says, "... I haven't yet read or heard any [explanations why minus times minus is plus]... not any I could totally accept, there was always some trick 'then we do like that'... to be totally realistic, I would get around it... I would simply say: that you must swallow."

The mathematics content of mathematics teacher education courses is the site of battles in many countries and, in a different domain, evidence of the beliefs about mathematics and about the teaching of mathematics.

> In most frameworks used to analyze teachers' professional knowledge—which implicitly serve as a basis for the structural organization of pre-service teacher education programs in Brazil—this knowledge is partitioned in such a way that content knowledge, usually identified with academic mathematics, assumes the status of fundamental component. Other components (didactical, pedagogical, curricular, pedagogical content knowledge, etc.), though important, are viewed essentially in relation to activities aiming for the "transmission" of the fundamental knowledge. It follows that subject-matter preparation for the mathematics schoolteacher has been conceived as an autonomous process, aiming basically to promote an internalization of the values, techniques, methods, conceptions and ways of thinking proper to academic mathematics. Thus, academic mathematics tends to occupy the center of gravity of the teacher education process, subtly pushing the discussion of issues related to teaching practice to the margins of the content courses (Moreira & David, 2005).

In terms of analysis of the responses of students to these various forms of data collection, Gadanis & Namukasa (2005) offer the following:

The analysis of the data from the 2004 elective Mathematics Course revealed a number of themes. These themes are briefly discussed below.

Frustration: "I'm still frustrated by my inability to see the conclusion or the point. I can't seem to push my thinking beyond the exercise to the solution, on my own."

Attention and insight: "I had a lot of moments where things just popped!"

Collaboration: "I felt really comfortable working in my group. It is easy to experiment with different things, and more ideas seem to come out."

Time: "I liked that we were asked what other methods can we come up with to test right-handedness/left etc. Then we were given time in class to go through and actually try ideas—it's been so long since I've had an experience like that in school. It was relaxing."

The complexity of mathematics: Most elementary mathematics teachers view mathematics as a subject of procedures for getting correct answers. As the Math Therapy course progressed, many pre-service teachers started expressing more elaborate views of mathematics.

Mathematics as a human activity: "Math has started to consume my thoughts."

Teaching mathematics: "I faced my 'math demons' and actually grew to enjoy a subject I thought would be my nemesis forever."

Beliefs and practice: "This class has completely shattered my understanding of math and how to teach math. It makes me feel that teaching math is going to be difficult—or at least more challenging than I previously thought. There are so many ideas—I feel overwhelmed" (Gadanis & Namukasa, 2005).

These categories, amongst others that researchers have produced, can provide a framework for research and development of teacher education programmes. Clearly the beliefs and experiences of student teachers who will be teachers of young children are quite different, in general, from those who will be secondary/high school teachers. The former are more likely to express the kinds of views captured in the Gadanis & Namukasa (2005) research quoted previously; other analyses of student teachers with a more secure knowledge of mathematics are provided by researchers. For example, on the basis of interviews, Proulx (2005) characterises five teachers as follows:

Future teacher's name	Perception and usage of the program
Albert, "The technician"	The program is seen as a source of potential teaching resources. It offered him, in his terms, some interesting and possible "tools" (activities, problems, good questions to ask) to use in his teaching.
Bertrand, "The mimic"	The principles/content brought forth in the program are considered optimal and ultimate: he does not question them and takes them for granted. The educators have an authoritative status for him and he "blindly" follows what was suggested.

Carl, "The self-assured teacher"	He recognized himself, as a teacher, in the principles brought forth in the program—involved implicitly in his practice. This program confirmed his practice and helped him to explicate (give a name to) the very practices he was enacting.
Donna, "The reflective practitioner"	The enunciated principles were seen as a philosophy of teaching, in which general ideas on education and mathematics teaching were the center. She did not focus on specifics for particular subjects, she aimed at themes like encouraging students to argue, working on diverse solutions, contextualizing mathematics, and so on.
Enrico, "The teacher in-action"	The program gave him a model in-action of teaching—not by the concepts brought forth in the program but from the way the educator was teaching. The educators were seen as teaching-in-action models.

3. Relations Between Attitudes and Beliefs "In Theory" and Practice

Much research has treated beliefs revealed in interviews and questionnaires, called "espoused beliefs", and beliefs as interpreted by observing teachers' actions, called "enacted beliefs", with the expectation that when teachers are consistent these will match. Researchers then try and explain why they are frequently very different. This approach is demonstrated by the assumption that using interviews and questionnaires reveals the presence of an identifiable object called a belief, or a system of beliefs, that is, the main determinant of a teacher's actions in the classroom. There are both theoretical and methodological problems here. Different contexts elicit different actions. Responses to interviews on the one hand and decisions one takes in a classroom on the other will certainly have significant overlaps, but they will not be the same, and to interpret the difference as a gap is to underestimate the nature of context in human activity and to perpetuate the mind—body distinction, that theories in the mind drive what one does in practice. On the contrary one could argue that those research techniques are productive of the findings. Further, there is circularity about the assumption of beliefs driving actions. Since beliefs are private and therefore hidden from the gaze of the researcher one can only infer a teacher's beliefs from her or his actions, including answering questionnaires or responding in interviews. One then claims that the actions are determined by the beliefs. Methodologically, then, one learns the most from research that succeeds in drawing together the teachers' practice and the teachers' views about that practice as close as possible, whether about mathematics itself or about mathematics education, if one can usefully make that distinction.

Based on three lessons for each of the five future teachers, I construed individual semi-structured interviews to delve into the intentions and background influences that framed these future teachers' classroom practices. The purpose of construing the interview on the basis of the future teachers' lessons was to ground and situate interview questions in the practices of the teacher.

> Such an effort was useful to better understand the future teachers' practices and rationales and to create interview questions that were contextualized and linked to those same practices—and not external to them (Proulx, 2005).

4. Systematic Observations of Classrooms

Theoretical frameworks to describe and categorise mathematics teaching are frequently developed from classroom observations, and there is clearly a need for the explication of the procedures used for developing those frameworks. That procedure may be developed from appropriate theories (e.g., Adler & Davis, 2006) or from a grounded approach.

> The purpose of the research reported in this paper was to develop an empirically-based conceptual framework for the discussion of the role of trainees' mathematics SMK [subject-matter knowledge] and PCK [pedagogical-content knowledge], in the context of lessons taught on the school-based placements. Such a framework would need to capture a number of important ideas and factors about content knowledge within a small number of conceptual categories, with a set of easily-remembered labels for those categories.
>
> This inductive process generated a set of 18 codes.
>
> We anticipate, however, that 18 codes is too many to be useful for a one-off observation. Our resolution of this dilemma was to group them into four broad, super-ordinate categories, or "units", which we term "the knowledge quartet".
>
> The first, foundation, consists of teachers' knowledge, beliefs and understanding acquired "in the academy", in preparation (intentionally or otherwise) for their role in the classroom. The key components of this theoretical background are: knowledge and understanding of mathematics per se and knowledge of significant tracts of the literature on the teaching and learning of mathematics, together with beliefs concerning the nature of mathematical knowledge, the purposes of mathematics education, and the conditions under which pupils will best learn mathematics (Rowland, Huckstep, & Thwaites, 2005).

The researchers exemplify the use of their tool for analysing teaching by examining one lesson of one teacher from their extensive data.

> Laura's professional knowledge underpins her recognition that there is more than one possible written algorithm for whole number multiplication. We

> conceptualise this within the domain of fundamental knowledge, being the foundation that supports and significantly determines her intentions or actions.
>
> It is perhaps not surprising that she does not question the necessity to teach the standard column format to pupils who already have an effective, meaningful algorithm at their disposal.
>
> At this stage of her career in teaching, Laura gives the impression that she is passing on her own practices and her own forms of knowledge. Her main resource seems to be her own experience (of using this algorithm), and it seems that she does not yet have a view of mathematics didactics as a scientific enterprise. (Rowland, Huckstep, & Thwaites, 2005)

5. Tensions and Challenges for Teacher Education

The goals of mathematics teacher education courses must be to develop the teaching potential of student teachers and improve the learning of mathematics in schools. Whether this is seen in terms of changing teachers' beliefs and/or their practices, inculcating a culturally valued set of teaching practices, or enabling student teachers to meet the demands of the regulatory system depends on the particular circumstances, ideals, and philosophies in different countries and in different institutions. The tension between recognising and supporting the developing teacher's autonomy and the teacher educator conveying her or his experience is at the heart of teacher education. "We are facing future teachers who have different visions and backgrounds and are interpreting things in different ways" (Bednarz & Proulx, 2005).

Indeed the research discussed previously, in eliciting students' beliefs and attitudes and analysing teaching practices, is aimed at identifying what might best be done in teacher education programmes.

> An initial knowledge base which I think it is a combination of a strong conceptual understanding of mathematics (SMK) and knowledge of a repertoire of representations (PCK) must be available to student teachers in pre-service teacher education. Otherwise their first students may well be led to think that mathematics is a complicated and unreachable form of knowledge because their teachers have not yet learned ways of communicating the subject (Amato, 2005).

Thus the direction of research on the changes student teachers experience through their pre-service teacher education courses are on their mathematics, their pedagogical practices, or both.

Improving student teachers' perception of mathematics as a subject matter involving exploration, pattern recognition, functions, problem solving, reasoning, modeling and applications, far beyond the "theorem-proof" activity typical of academic math courses.

Moreover, providing for a context in which future teachers can grasp the nature of mathematics culture, its beauty and its intellectual fulfillment so that they develop an enthusiastic attitude towards communicating these values to school children, has been a true challenge.

To meet these challenges, and similar ones in other areas of specialization, the Department of Education made it its departmental policy to include in the preparation program for high school teachers courses specially designed to bridge between the pure and applied subject-matter courses, and the psychology and methods courses taken towards a teaching certificate in any particular area.

The discussion will focus on values such as mathematical usefulness vs. mathematics as a human endeavor; motives for the development of mathematics; failure and success in mathematics; mathematics for the majority vs. mathematics for the elite—and more (Movshovitz-Hadar, 2005).

6. Concluding Remarks

These studies indicate that the research community has moved on substantially from the questionnaire as the lone tool for eliciting teachers' or student teachers' beliefs about mathematics and mathematics teaching and learning, observing in the classroom, and identifying a gap between espoused and enacted beliefs. The literature on activity theory and on social practices provides richer perspectives of identity formation and on the study of trajectories as people move into practices. Those trajectories differ from others because of prior experiences, goals, needs, and the tensions and conflicts that arise. This calls for researchers drawing on more sophisticated research tools such as those reported here. The study by Brown and McNamara (2005) has been mentioned; Ensor's (2001) analysis of mathematics student teachers' formation is another that draws on theory, in this case Bernstein's sociology, to provide principled accounts of students' voices and their identities. Looking to developing directions in the research community, one might expect, and indeed can already encounter, studies that analyse student teachers' experiences through their courses in terms of communities of practice (Kanes & Lerman, 2007) and of activity theory. The complexity is captured in the following: "In the novice teacher we see the very beginnings of a process of reconciliation of pre-existing beliefs, new 'theoretical' knowledge, 'practical' advice received from various quarters, in the context of highly-pressured, high-stakes school-based placements" (Rowland, Huckstep, & Thwaites, 2005).

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Chapter 1.2.2
School Experience During Pre-Service Teacher Education from the Students' Perspective

Merrilyn Goos, *University of Queensland, Australia*, and B. Arvold, N. Bednarz, L. DeBlois, J. Maheux, F. Morselli, J. Proulx

A challenge for teacher education is to understand how pre-service teachers learn from experience in multiple contexts—especially when their own schooling, the university methods course, and their practicum experiences can produce conflicting images of teaching. This chapter examines how pre-service teachers interpret their school experiences in the light of their university pre-service courses, their personal histories, knowledge, beliefs, and attitudes, and the specific constraints of the school environment.

1. Pre-Service Teachers' School Experience: (Dis)Connections with University Coursework

Ensor (2001) notes that the apparent disconnection between the teaching practices privileged by the university methods course and the practices adopted by pre-service and beginning teachers in their classrooms have been:

> variously attributed to educational biography (e.g., Lortie's [1975] "apprenticeship of observation"), school setting (e.g., Zeichner & Tabachnik, 1981), differential engagement by students with their teacher education courses (e.g., Lacey, 1977), or a failure to change teachers' belief systems (Cooney, 1985; Thompson, 1992) (p. 316).

Ensor acknowledges these influences in her longitudinal study of the transition from pre-service to beginning teaching of secondary-school mathematics, but she argues that the theory-practice disjuncture is better understood in terms of novice teachers' recontextualisation of pedagogical practices from the pre-service course to their school experience. This recontextualisation is shaped by access to recognition rules, which allow beginning teachers to identify and discuss "best practice", and realisation rules, which enable them additionally to implement best practice in their classrooms. Ensor suggests that university methods courses may provide pre-service teachers with access to recognition rather than realisation rules, leaving them unable to develop the repertoire of practices promoted by teacher educators.

Contributions to the 15th ICMI Study drew on a variety of theoretical positions to investigate ways in which pre-service teachers make the transition from university to classroom. Canadian research carried out by DeBlois and Maheux (2005) adopted

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the theoretical framework surrounding situated cognition (Brown, Collins, & Duguid, 1989) to account for the influence of teacher education students' experiences in different contexts—as school pupils, university students, and pre-service teachers in a school setting. This research investigated gaps between primary pre-service teachers' planning and enactment of classroom activities, their explanations for these gaps, and the nature of the learning that resulted. Discussion teams were formed to promote joint reflection, each consisting of the pre-service teacher, his or her partner teacher, the school's special education teacher and a researcher. Meetings of the teams were held before and after the pre-service teachers tried out planned activities in the classroom.

Analysis of transcripts of the team discussions resulted in the identification of four types of adaptations to describe gaps between plans and classroom enactments: projective adaptations, withdrawal adaptations, normative adaptations and avoidance adaptations. Projective adaptations occurred either at or near the beginning of activities, whenever pre-service teachers exploited pupil attitudes or interest "to question them further" or whenever they used "a pupil's difficulty or explanations to foster or pursue a discussion". Withdrawal adaptations occurred when pre-service teachers deemed "the pupils capable of resolving certain difficulties" and decided not to intervene. This sometimes led them to turn the discussion over to the pupils or allowed them to observe consistent error patterns amongst several pupils. Normative adaptations occurred whenever pre-service teachers observed "a gap between a pupil's comments and the teacher's expectations", and prompted the pupil to adjust or adapt by pointing out the error. Avoidance adaptations arose whenever pre-service teachers simplified the task at hand or lowered their expectations, for example, in response to pupils' lack of motivation or understanding.

The number and type of adaptations deployed appeared to depend on three factors. The first relates to the specificity of the teaching intention, possibly stemming from the pre-service teacher's previous experience as a university student. Well-defined intentions seemed to prompt the pre-service teachers to adjust to pupil reactions projectively, while ill-defined intentions prompted them to adhere more closely to their plan even when this was not appropriate. The second factor is concerned with the level of comfort with pupil reactions, teaching materials and mathematical concepts, and preferred teaching approaches, which may stem from growing experience as a pre-service teacher. Comfort is associated with projective adaptations and discomfort with a need for control, evidenced by normative or avoidance adaptations. The third factor is based on pre-service teachers' previous experience as a school pupil, in which a given event is posited as resulting from a "best" intervention. As a result of these team discussions, the pre-service teachers were better able to understand and justify their choices with respect to interpretations of pupil reactions. They—and the teacher educators—were then able to recognise the evolution of the teaching situation (Brousseau, 1983), the "taking charge" of classroom activities, and the pre-service teachers' projection into their professional practice.

In another Canadian contribution to the 15th ICMI Study, Bednarz and Proulx (2005) investigated the extent to which secondary pre-service teachers appropriated aspects of their university mathematics education program. They invoked

the concepts of "action knowledge" (Schön, 1983) and "structuring resources" (Lave, 1988) as principles guiding the design of a four-year secondary mathematics teacher education program that focuses specifically on the learning of mathematics teaching-in-action rather than learning-about-action. From the pre-service teachers' perspective, emphasis is placed on actual teaching situations in a cycle of planning based on a conceptual analysis of a mathematical notion, a prior analysis of curricula and usual teaching approaches, the construction of a repertoire of chosen problems, a classroom teaching experiment consisting of a sequence of lessons, presentation of a reflective analysis of these lessons to fellow students and an experienced practising teacher, and subsequent re-adjustment of plans for the lessons taught.

Bednarz and Proulx summarised case studies of five pre-service teachers' practices that reveal interesting differences in how they appropriated elements of this teacher education program. Analysis of interviews showed that each had a different view of the program (**Table 1.2.2.1**); these views were also reflected in their video-recorded teaching practices.

Enrico, Donna, and Carl appropriated principles on a general level: for them teaching is a way of being rather than doing. Albert worked mostly on a pragmatic level, using ideas from the course only in a technical way and at specific moments, while Bertrand reproduced what he was taught in the program without really knowing why. Each of these pre-service teachers used their particular "lens" to interpret many other aspects encountered in their school practicum, for example, when they interpreted textbooks, interacted with practicum supervisors and associate teachers, analysed final exams, and so forth. These results prompted Bednarz and Proulx to question their own practice as teacher educators modelling "good practice" and acknowledge the problematic nature of the relationship between pre-service teachers' university and school experiences.

Table 1.2.2.1 Pre-service teacher views of the teacher education program (Bednarz & Proulx)

Albert	The program is seen as a source of potential teaching resources. It offered him, in his terms, some interesting and possible 'tools' (activities, problems, questions to ask, etc.) to use in his teaching.
Bertrand	The principles/content brought forth in the program are considered optimal and ultimate: he does not question them and takes them for granted. The teacher educators have an authoritative status for him, and he 'blindly' follows what was suggested.
Carl	He recognized himself, as a teacher, in the principles brought forth in the program—involved implicitly in his practice. This program confirmed his practice and helped him to explicate (give a name to) the very practices he was enacting.
Donna	The enunciated principles were seen as a philosophy of teaching, in which general ideas on education and mathematics teaching were the centre. She does not focus on specifics for particular subjects, she aims at themes like encouraging students to argue, working on diverse solutions, contextualising mathematics, and so on.
Enrico	The program gave him a model-in-action of teaching—not by the concepts brought forth in the program, but from the way the teacher educator was teaching. The teacher educators were seen as teaching-in-action models.

Reflecting on the enormous variation in the pre-service teachers' perceptions of the university program led Proulx (2005) to further question the structure, development, and possible objectives of mathematics teacher education programs:

> Drawing on Pimm's (1993) concept of "change merchant", Breen (1999) explains that for some educators it has become a central task to convince others of the quality of their own particular merchandise and have people use their "magical" infallible method—that is, to have the intention of controlling and of striving towards creating or generating "perfect teachers".

However, Proulx suggests that the outcomes of a teacher education program cannot be controlled, as they are more diverse and unpredictable than we might expect. This is not to say that it is unproductive to educate teachers or that there cannot be specific goals in a mathematics teacher education program. The issue, rather, is how to treat the notion of objectives.

Oriented by the research results outlined above, Proulx offers a redefinition of "objective" as a starting point for development instead of an end point to attain. From this proposal he theorises that objectives could be framed in terms of expanding the space of the possible (Davis, 2004). The focus would then shift from ideas of conformity and convergence on a specific way to teach towards ideas of emergence and expansion, thus striving for legitimisation of a generative model of teaching that views teachers as responsible and autonomous beings. The significance of such a model is that teachers should "come to possess rationales and reasons to support the actions and claims they make in the classroom". Proulx argues that this is where the identity of the teacher stands: in the acquisition and construal of a personal, defensible, sustained rationale (stance and position).

2. Pre-Service Teachers' School Experience: Personal Histories and Embedded Traditions

Research has suggested that pre-service mathematics teachers' beliefs, orientations toward knowing, goals and aspirations, and early educational experiences are critical to teaching practices. Morselli (2005) claims that the situation for prospective primary school teachers is of special concern as their previous experience of learning mathematics at school has not always been positive. Her contribution to the 15th ICMI Study adopted Charlot's (1997) concept of relation to knowledge (*rapport au savoir*), of which the relation to mathematics (*rapport aux mathématiques*) is a special case. The relation to mathematics is defined as "the set of relationships that the subject has with some objects (theorems, activities, but also people, situations, events) that are related to mathematics". Charlot suggests an analysis of the difficulties of students in terms of their interpretation of school experiences. As a consequence, Morselli takes into account the personal histories of pre-service primary school teachers in asking, "How can the concept of relation

to mathematics help to better understand the needs and difficulties of pre-service teachers?"

Although Morselli's study does not examine participants' school experience during the teacher education program, it does consider possible relationships between their past and present relation to mathematics and their future work as teachers of mathematics. A questionnaire seeking this information was completed by 122 pre-service primary teachers. Responses to open questions gave personal opinions about mathematics referring to school life, teachers, and difficulties in learning mathematics, thus confirming the centrality of school experiences. Responses to the question, "How do you feel, if you think of your future job as a teacher, referring in particular to the teaching of maths?" were varied: 12 percent felt optimistic and prepared, and 31 percent affirmed their hope that they would help their pupils like mathematics. On the other hand, 14 percent felt worried, 3 percent feared they would make their pupils hate mathematics, and 15 percent doubted that they would be able to make their pupils understand. These answers suggest that their relation to mathematics may influence the quality and efficacy of their teaching.

In a related contribution to the 15th ICMI Study, Arvold (2005) uses the concept of embedded tradition to explain inconsistencies between teacher education programs and the classroom practices of pre-service and beginning teachers. She proposes that embedded tradition, an amalgamation of beliefs, goals, orientations towards knowing (and thus reminiscent of Morselli's "relation to mathematics"), and other constructs that are closely aligned with each other and central to a person's identity, can provide a springboard for personal and professional growth.

Nine cohorts of pre-service secondary mathematics teachers participated in Arvold's research, and many volunteered to continue their involvement after graduation. Through an early study of three novice teachers, Arvold came to realise that they attended to different aspects of the pre-service program and made sense of the program quite differently as well (cf. Bednarz & Proulx, 2005). Their orientations towards knowing and their beliefs about mathematics, teaching, and learning were different, but most striking after analysis of data from their year in the program and their first year of teaching was the influence of their differing goals and aspirations. Follow-up visits to these teachers' classrooms near the end of their eighth year of teaching confirmed that their goals and teaching methods remained much the same. The question emerging from working with these initial participants was, "Could we better prepare teachers if we focused on helping them springboard from their embedded traditions?" Arvold's subsequent research has been in response to this question. She challenges us to reconsider our goals as teacher educators and to ask whether we are preparing students well if we only prepare them to model what we demonstrate. Her work strongly suggests that:

> teacher educators need not direct all teachers to what they consider successful methods, i.e., 'best practice', but instead teacher educators might at least consider helping teachers use their embedded traditions to springboard themselves into a profession in which diversity of teaching methods will provide students with even greater chances for mathematical success.

3. Learning From Experience: School Contexts, Personal Histories, and University Coursework

Rather than examining separately the disconnections between pre-service teachers' school experiences and their university coursework, personal histories, beliefs, goals, and orientations to knowing, Goos's (2005) contribution to the 15th ICMI Study outlined a theoretical framework for simultaneously analysing these relationships and how they shape novice teachers' professional identities. The framework draws on sociocultural theories that interpret learning as increasing participation in social practices. It extends Vygotsky's (1978) concept of the Zone of Proximal Development (ZPD) to enable analysis of teachers' interactions with their environment and other people, over time and across different contexts, by introducing two additional zone concepts originally proposed by Valsiner (1997). The Zone of Free Movement (ZFM) represents environmental constraints within the school context, such as student characteristics, curriculum and assessment requirements, and availability of teaching resources, while the Zone of Promoted Action (ZPA) symbolises the efforts of a teacher educator or more experienced colleague to promote particular teaching skills or approaches. Pre-service teachers develop under the influence of two ZPAs—one provided by their university program, the other by supervising teacher(s) during their school experience—which do not necessarily coincide.

Vignettes from a case study of transition from pre-service to beginning secondary teaching illustrate how the framework can guide analysis of teachers' professional experiences (**Fig. 1.2.2.1**).

The ZPA offered by supervising teachers in Sandra's practicum school was not a good match for the ZPD that defined her knowledge, beliefs, and goals about using technology in mathematics education, nor did it provide a pedagogical model consistent with the technology emphasis of the pre-service course. While some elements of Sandra's ZFM (e.g., easy access to calculators that no one else wanted to use) presented favourable opportunities to use technology, others (e.g., students' attitudes and lack of motivation) may have acted as constraints, discouraging her from using technology again. Sandra's response to this configuration of experiences suggests that there was sufficient overlap between the university course's ZPA and her personal ZPD for her to continue enacting her pedagogical beliefs about using technology.

Compared with her practicum experience, Sandra's first year of teaching offered a more expansive ZFM. Yet there was no ZPA within her school environment, and lack of professional development opportunities in this isolated community made it difficult for her to access an external ZPA. While she was still able to draw on the knowledge gained during her university program (the pre-service ZPA), Sandra recognised her need to gain new ideas via collaboration with other more experienced teachers beyond the school in order to further develop her identity as a teacher for whom technology was an important pedagogical resource.

Goos proposes that the zone framework could support pre-service teachers' learning by helping them to analyse their school experiences (ZFM), the pedagogical models these offer (school ZPA), and how these experiences reinforce or

Vignette #1: School experience

Sandra's university methods course emphasised integration of technology (computer software, Internet, graphics calculators) into mathematics teaching and learning. Her responses to questionnaires concerning mathematical and pedagogical beliefs demonstrated strong commitment to use of technology as part of a student-centred teaching approach. Her practicum placement was in a large school with many computer laboratories that had recently purchased its first class set of graphics calculators. None of the teachers had found time to learn how to use the calculators. Sandra was familiar with computer software used in mathematics teaching and regularly searched the Internet for teaching ideas. She used the computer laboratories in her mathematics teaching during the practicum, but she had not observed other teachers in the school use any kind of technology with their classes. When Sandra was teaching linear programming, she decided to show her mathematics class how to use the graphics calculators because this would help them understand how to plot the objective function, observe the feasible region, and find the optimal solution. She found and adapted an optimisation activity from the Internet for this purpose. Because the students had never used graphics calculators before, she also devised a worksheet with keystroke instructions and encouraged students to work and help each other in groups. Yet she encountered strong resistance from the students, which seemed to stem from their previous experiences of mathematics lessons. Other teachers focused on covering the content in preparation for pen and paper tests and did not allow the students to work in groups. The students were not interested in helping each other or in learning how to use technology if this would be disallowed in assessment situations.

Vignette #2: First year teaching

After graduation Sandra moved to a small school in a rural town far from the city. The school had several class sets of graphics calculators as well as a hire scheme for senior students, but no other mathematics teachers knew how to use them effectively. Sandra had used all of the technology-based resources provided in the university methods course and continued searching for more. But she found this difficult because of a slow Internet connection and lack of access to professional development. Nevertheless she remained enthusiastic about using technology and could describe the benefits for students' learning in terms of developing deeper understanding of mathematical concepts. She used email to maintain contact with colleagues in other schools as a source of teaching ideas.

Fig. 1.2.2.1 Vignettes from a case study of transition from pre-service to beginning teaching (Goos, 2005)

contradict the knowledge gained in the university program (university ZPA). This analysis could also support the transition to the early years of teaching by promoting in beginning teachers a sense of individual agency within the boundaries and constraints of the school environment (ZPD within ZFM).

4. Concluding Remarks

Each of these studies speaks to the need for teacher educators to help pre-service teachers develop their own professional identities and acknowledges that identities are shaped by complex and often unpredictable interactions between pre-service teachers' past histories, their school experience during the teacher education program, and the objectives and practices promoted by the university methods course.

Modelling of "best practice" during the pre-service course does not lead to the formation of "perfect teachers" cast entirely in the mould of the teacher educator. This is indicated by researchers' use of terms such as "recontextualisation" (Ensor, 2001), "appropriation" (Bednarz & Proulx, 2005), and "adaptation" (DeBlois & Maheux, 2005) to describe ways in which pre-service teachers draw on their university methods courses. Rather than viewing these disconnections—between theory and practice, university and school, plans and their enactment, teacher educators' objectives and pre-service teachers' aspirations—as a problem to be solved, it may be more helpful to treat the school experience component of the teacher education program as an alternative, and productive, site for engaging with pre-service teachers' identities within the profession.

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Chapter 1.2.3
First Years of Teaching

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1. Introduction

A recurring and crucial theme in research on teacher education is the relation between pre-service teacher education and the day-to-day practices of mathematics teachers in schools. As we have seen in previous chapters, this relation appears already when pre-service teachers are engaged in practice periods, which in many cases form part of their teacher education. However, it appears also, and sometimes more acutely, in the experiences of teachers in the first years after completing teacher education. The first years of teaching practice may indeed be viewed as a period of transition at several interrelated levels:

- at an *epistemological* level, for example, when it appears necessary to adapt the (sometimes quite "academic") forms of knowledge acquired in pre-service education to the conditions and requirements of teaching;
- at an *institutional* level, passing from one institutional context (e.g., a university) to another one (a school, usually within a wider system of schools), often with quite different norms and other cultural assets; and
- at a *personal* level, the change from being a student in a community of students to being a professional in a community of teachers. This clearly depends on the previous aspects but also on more contingent conditions and personal beliefs.

In this section we shall take a closer look at the phenomena of transition as they occur and are studied in contributions to the 15th ICMI Study as well as in related literature. As we shall see, the institutional transition is organised quite differently in different countries and educational systems (even within one country), and this clearly is important in understanding different conceptions, in these contexts, of the other two levels. Britton, Paine, Pimm, and Raizen, (2003, p. 16) define a system of induction as a coherent set of policies and practices set up by a country (or another official entity, above the level of single institutions) to help new teachers in their practice. While most of the contributions consider just a single such system, we may also see the effects of different induction systems by looking at the studies together. Indeed, they can be considered a part of teacher education in the broader sense of frameworks for developing teachers' professional knowledge (see Tatto, Paine, & Schwille, 2005).

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2. The Epistemological Transition

Several models are proposed in the literature, and also in this ICMI study, to shed light on the complexity of the professional knowledge which teachers need to develop in order to teach mathematics in required (e.g., by official regulations) or otherwise desirable ways. Bergsten and Grevholm (2005) point out two possible "divides" which may occur in the teachers' knowledge: between disciplinary (mathematical) and pedagogical knowledge, and between the practical and theoretical parts of each of these. Drawing on Chevallard's (1999) anthropological theory of didactics (see Barbé, Bosch, Espinoza, & Gascón, 2005, Sect. 2) Bergsten and Grevholm (2005) point out the "ideal" intimate connection between theoretical and practical blocks of knowledge as an organisation system for practice: on the one hand, theoretical blocks of knowledge are needed for explaining, structuring, and giving validity to work in the practical block, while on the other, the theoretical block has no meaning if detached from the practical block (tasks and techniques) it explains, structures, and justifies. They then go on to point out the lack of a unified scholarly based organisation of pedagogical knowledge, in particular the "diverse" and perhaps altogether underdeveloped theoretical block of such knowledge. Without a relation between mathematical and pedagogical knowledge at the level of theory blocks, "teachers are trained to stay at the 'punctual level' (i.e., limited to particular problems and techniques only) at their future work in classrooms, giving way for an instrumental way of communicating knowledge with their pupils" (ibid.). This situation is illustrated, for example, by the case study described in detail by Barbé et al. (2005), where the incoherent punctual organisations of mathematical practice found in the upper-secondary Spanish classroom are also related to more fundamental inconsistencies in the curriculum. Even if the teacher has sufficient knowledge of the academic mathematics behind the curriculum he teaches, the lack of a coherent, corresponding, and articulated pedagogical organisation of knowledge leaves him with few possibilities to avoid this instrumental approach in the classroom.

A similar divide between academic mathematics and school mathematics (the latter being understood as validated knowledge specifically associated with the development of school education in mathematics) is pointed out by Moreira and David (2005), who study the particular case of the construction of rational and real numbers as they are conceptualised within these two organisations of knowledge. According to them, the formal presentation of rational numbers found in "academic mathematics courses is ultimately dissonant with the forms of knowledge about real numbers found in school teaching practice since it ignores many important pedagogic issues, such as the rationale for extending ... the notion of number". In case the beginning teacher has mainly been presented to the academic mathematics knowledge organisation—which seems to be the case in the teacher education programme the authors describe—she may then be left to construct such rationales as she best can or to resort to an instrumental approach to teaching in which rationales and coherence are more or less absent.

In fact, the balance—and possible integration—between "academic" mathematics and "school mathematics" in various forms seems to be a concern in several papers presented in this ICMI study. It can be more or less left to the beginning teacher depending on the organisation of her pre-service education and the system of induction (see the next subsection).

Ball, Bass, Sleep, and Thames, (2005) present what they call a "practice-based theory of mathematical knowledge for teaching", consisting of:

- common content knowledge, the mathematical knowledge of the school curriculum (essentially what is aimed at for students);
- specialized content knowledge . . . mathematical knowledge that teachers use in teaching that goes beyond the mathematics of the curriculum itself (and could be needed, e.g., to analyze errors made by students);
- knowledge of students and contents, for instance, about what mathematics students find interesting or challenging and about what students are likely to do with specific mathematical tasks; and
- knowledge of teaching and contents, such as knowledge about instructional sequencing of particular contents, about useful examples for highlighting salient mathematical issues, and about advantages and disadvantages of representations used to teach a specific content idea.

In their session, the authors invited participants to analyse how these knowledge components were needed in classroom teaching, based on video material, and how they differed. They also recognize that the mathematics knowledge it takes to teach it is inadequately understood at present and, one may speculate, accordingly difficult for teachers to develop in their first years of practice, as it would presumably also be taught inadequately in teacher education.

In several studies (e.g., Ma, 1999; Stevenson & Stigler, 2000) the practices of East Asian teachers—such as collaborative "lesson study"—have been pointed out as ways in which teachers, and in particular new teachers, may develop these and similar forms of school mathematics knowledge during their professional lives, through collaborative design and discussion of individual mathematics lessons (with concrete mathematical target knowledge). Teacher development through the Japanese model for lesson study is the topic of several contributions to this ICMI study, such as the work session by Robinson (2005) on lesson study in Israel and the work session by White on lesson study in Malaysia and Australia.

In his paper, Li (2005) describes Chinese teacher education as being largely focused on academic mathematics, in this sense, similar to the Brazilian case described by Moreira and David (2005) and adds, towards the end of his paper, that "Chinese teachers . . . develop profound understanding of fundamental mathematics during their teaching careers". It seems from this contention, as well as from studies such as those by Ma and Stevenson & Stigler, that initial training may not have to take full responsibility for new teachers' development of the elements of school mathematics knowledge described above. This seems to concord with Butlen's conclusion in his contribution on the specialised knowledge which is required to teach

mathematics in disadvantaged areas in France: "Initial training cannot tackle all the specific issues that might pose problems for new beginners; it must therefore be resumed and developed during the early years in post" (Butlen, 2005).

However, perhaps because the opportunities for doing so in pre-service education are typically less developed in most Western countries (see the next subsection), he also says that it is "indispensable to integrate them [knowledge components related to mathematics teaching in schools with many children from disadvantaged backgrounds] into a specific initial training which includes arrangements adapted to analysis of teaching practices". The distribution of responsibilities between pre-service education and induction during the first years of teaching seems to be a crucial issue for resolving some of the problems for beginning teachers, and it cannot be considered solely as a question of improving our understanding of the problem from an epistemological point of view. It involves, in a salient way, the institutional transition.

3. The Institutional Transition

In the anthropological theory of didactics (Chevallard, 1999; Barbé et al., 2005), teaching and learning are viewed as an activity situated in an institutional setting (Bergsten & Grevholm, 2005), that is, the corresponding practical and theoretical knowledge is fundamentally anchored in institutions as an institutional ecology of knowledge (Chevallard, 1992). In particular, knowledge about mathematics teaching tends to take on more theoretical and disciplined forms when developed, taught, and learned within an academic institution than when it is developed and used within a school.

A shift from seminar (non-academic college) to university-based teacher education has taken place in many countries over the past thirty years in some form and at some levels at least. Bergsten and Grevholm (2005) point out that this may have entailed a widening of the gap between practice and theory blocks of teacher knowledge while on the other hand enabling a more systematic, if not scientific, approach to the knowledge underlying the teaching profession.

> The seminar tradition, by its experience-based mode of reference, is both oriented and constrained to practical blocks, in the disciplinary as well as in the pedagogical realm ... It is normally only within mathematics and pedagogy as university disciplines that the theoretical levels of its knowledge are discussed ... These disciplines live in different departments at the university with normally little or no interaction.

After examining briefly how this division has developed in Sweden, the authors conclude that

didactical research is needed to develop a relevant theoretical block to merge the didactic divide . . . one of the major goals for research in mathematics education is . . . the development of a body of educational knowledge in mathematics, to make teacher education an institution able to work in line with the professional competence paradigm for what it means to be a mathematics teacher in school.

The authors thus call for better alignment of the knowledge development that takes place within universities and schools, and this is set out as a special task for the didactics of mathematics, or mathematics education research, as a third disciplinary component besides mathematics and pedagogy. It must be noted here that this third discipline, as an academic one, is relatively new and has developed in different ways in different institutional contexts. Many mathematics educators are, or have been, active teachers within a primary or secondary school, as is evidenced by the broad range of professional experiences and background that are evidenced by the contributions to this study.

In fact, the direct implication of mathematics education researchers in development projects within schools could be one important venue for creating the institutional alignment mentioned previously and hence to construct a smoother transition for beginning teachers as they move from university to school. The contribution of Wood (2005) is an example of such a project which had as its goal to investigate how primary-school teachers learn to develop their classroom teaching in accordance with reform schemes in the United States. It involved six primary-school teachers in their second year of teaching. Following the suggestions of a researcher, the teachers adopted a three-step model for collegial development devised by Jaworski (1988) involving reflection upon video recordings of their teaching and comparing these to a written record of their plans and expectations prior to the lesson. They were then to devise a "plan of action" to carry out in the classroom based on the results of their reflections. Notice that the elements of the reflection procedure implemented by Wood and her co-researchers bear at least some resemblance with the long-established format of lesson study in Japan (e.g., Stevenson & Stigler, 2000, Chap. 9). However, in lesson study, teachers only occasionally interact with researchers, and more importantly, the design, reflection, and redesign is done collectively rather than individually.

Wood's paper focuses on how each of the teachers learned from this process. While all of the primary teachers began by relying on two models for instruction, a model of students' behavior during instruction and a model of themselves as teachers of mathematics, only some of the teachers were able to create more complex forms of practice, in line with the constructivist views of learning set out by the American reform scheme. These students were characterised by building essentially new models of the ways students made sense of mathematics and then realising a need to transform their ways of teaching. On the contrary, other teachers focused extensively on students' behaviors and external factors and continued to

use the same form of teaching. The experiment thus confirms the need for teachers to go beyond pedagogical schemes for student behaviour, to reflect systematically on students' mathematical thinking in the specific context of a teaching situation. The study also seems to suggest that explicitly asking them to do so may not in itself suffice. However, clearly the existence of frameworks for engaging in such reflections in connection with professional practice could be important, not least for beginning teachers, and they seem to be lacking in many environments (see also the case of Sandra at her rural school in Australia, reported on in the previous chapter).

More generally, there is a huge variation, among systems of education, when it comes to official regulations for the institutional transition from university to school—if they exist at all. In some cases (like that of Sandra) the new teacher is left practically on her own in her practice right after graduation, while in other countries (like China; see Ma, 1999, 137f, as refered to in Li, 2005) a system of mentorship is set up to accompany the new teacher during the first years of teaching. Britton et al. (2003) present study-induction systems in different geographical sites (France, Japan, New Zealand, Shanghai, and New Zealand). In all of these places, they found comprehensive and official frameworks for supporting new teachers in their first years of teaching. There are many and striking differences between these induction systems. In particular, the university of graduation assumes quite different roles within induction processes. For instance, in France, induction is highly standardised (see Sayac, 2005): teachers take up a part-time teaching job along with finishing their last year of teacher education and receive visits and guidance at their school from a designated university-employed mentor. In New Zealand, a variety of support providers are available to the beginning teacher, mainly within the school of employment; the authors mention that some schools achieve the goals of the national induction policies better than others (Britton et al., 2003, p. 191). It appears from the studies in Britton et al. (2003) that induction systems could be very important for the experiences and development of teachers in their first years. However, it seems to be work for the future to undertake a more extensive international comparative study of induction systems.

4. The Personal Transition

Case studies of individual teachers' experiences during their first years of teaching can be used to study more deeply the effects of institutional and epistemological constraints in the teacher profession and in particular the way into it. Small-scale qualitative studies of individual teacher experience are reported on in several contributions to the ICMI study, and some of the cases are about beginning teachers. However, given the short format for contributions, we have only outlines of the details. For instance, the vignettes of the case of the teacher Sandra, which were presented in the previous chapter, are obviously a quite condensed version of her experiences: a lack of opportunities for professional development in a community of practice.

The phenomenon of isolated teacher practice is found in many other studies of beginning teachers, such as Skott (2001), who studies the school mathematical beliefs and practices of a Danish novice teacher who seems to develop these in a similar state of isolation. This may not necessarily be viewed as a problem by the "culture" of teaching:

> Traditionally, Danish teachers have been allowed a considerable degree of freedom with regard to their choice of teaching methods. Combined with loose descriptions of the mathematical contents in the national curriculum . . . and with a weak tradition for collaboration among teachers in each subject, individual teachers are given and left with considerable influence and responsibility in their classroom (Skott, 2001, p. 9).

Stigler and Hiebert (1999, p. 123) mention "the often described isolation of U.S. teachers". The independence of the individual teacher is seen as positive by many teachers, but the authors contend that collaborative research-and-development systems for American teachers—such as those found in Japanese lesson study—would be required to achieve "steady, continuous improvement of teaching" in the United States (p. 127).

Skott (2002) studied another case of how a young teacher may experience a fundamental gap between the ideals and modalities adopted in teacher education, and the constraints and opportunities found (or perceived) in a real-life school. "Here, not only the relative isolation in which the new teacher works, but also the pressure he is under . . . to ensure that the students perform well in the next test" (p. 215) contributes to the teacher's experience that a number of the ideas learned and adopted in pre-service education are just not viable in teaching. This in particular seems to affect possibilities for enacting constructivist principles for teaching.

It seems obvious that the needs for collaborative frameworks are particularly important for the beginning teacher. Teacher students in a university will often be treated to collaborative forms of work, both on theoretical and practical projects, as evidenced by several chapters of this volume. For instance, Gómez (2005) describes how a methods course promoted collective learning for teaching by aiming at constituting a kind of community of practice among the students. One may imagine the disenchantment of new teachers who, after graduation and being hired at a school, are to face the "influence and responsibility" of teaching their classes without substantial interaction with other teachers on how to do so.

5. Concluding Remarks

The first years of teaching can be seen as a transition with many interdependent components: from being a teacher student in a university environment, where mathematics and teaching is often considered in more theoretical ways, to a (more or less autonomous) status of being a professional in a school, in charge of a number of practical problems related to teaching and school mathematics. We have considered a mosaic of studies from different parts of the world which, in various ways, shed light on these transitions. The international perspective seems very important

here, as it may help us to reject the fatalism that often results from a perspective which is confined to a single system of education—where, as one says, "It has always been like that". Needless to say, systems of schooling—including teacher education—display a surprising level of inertia. Looking beyond them may help us recognize that their defaults are not inevitable. This seems in particular to be the case for some of the problems faced by beginning teachers, including isolation and lack of resources for professional growth as a mathematics teacher.

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Theme 1.3
Mathematics Educators' Activities and Knowledge

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Teacher educators are important in mathematics teacher education. They are responsible for designing and developing mathematics teachers' learning experiences. Research in mathematics education is beginning to recognize the relevance of exploring and reflecting on mathematics teacher educators' activities and knowledge. That is why this topic is included in this book, even though there were no papers on the subject from the study conference participants. The theme is organized in four papers referring to different aspects of mathematics teacher educators' roles, knowledge, and learning.

In the first chapter, "Mathematics educators' knowledge and development", Zaslavsky suggests parallelisms among the learning processes of mathematics teachers, mathematics teacher educators, and mathematics teacher educator educators. She highlights the role of tasks for promoting learning and of the educator as facilitator. Since there are no teacher educator education programs, teacher educators learn by reflecting on their own experiences. Learning should emerge in the interplay of the teacher educators roles: as a researcher, as a facilitator, and as designer of tasks.

Pope and Mewborn's chapter, "Becoming a teacher educator: Perspectives from the United Kingdom and the United States", describes several aspects of educators' training and of becoming a teacher educator in the United States and the United Kingdom. They describe who becomes a teacher educator, what the tasks and duties of a teacher educator are, and how the training of teacher educators is being supported in both countries. They put in evidence, with the comparison of two countries, the great variety across and inside countries concerning how to become a mathematics teacher educator.

Chapman's chapter, "Educators reflecting on (researching) their own practice," is a clear and well organized literature review on mathematics educators reflecting and researching their own practice. She analyzes a set of studies in which educators systematically explore their teaching approaches taking into account all of the students in the educator's class. There are few such studies. She approaches the questions of what and how did they research, and what can be learned? There are several difficulties involved in this type of research. For instance, context is important and, therefore, one cannot generalise easily. Nevertheless, the point is that mathematics educators can and should learn from experience in order to better promote students' learning.

In the final chapter, "Educators and the teacher training context", Millman, Iannone, and Johnston-Wilder suggest some ideas that can be valued by mathematicians and mathematics educators as issues on which collaboration can develop: the concept of the Knowledge Quartet, the notion of mathematical habit of mind, and the comparison of ways in which mathematicians present fractions to future teachers. They also present preliminary results of their international survey on mathematics and pedagogy content courses and the role of mathematicians and mathematics educators on them.

Chapter 1.3.1
Mathematics Educators' Knowledge and Development

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This chapter focuses on mathematics educators. Generally, mathematics educators include educators who facilitate the learning of mathematics, as well as educators who facilitate the teaching of (or learning to teach) mathematics. There is much in common between the knowledge and development of mathematics teachers and that of their educators, yet there are some distinctive characteristics of teacher educators' knowledge which are worthwhile to examine (Zaslavsky & Peled, 2007).

1. Models of Educators' Development

Zaslavsky & Peled (2007) point to a number of trends in recent years with respect to studies related to teacher education. One of these trends has to do with entering mathematics teacher educators into the picture. Researchers have started to conceptualize teacher educators' role and knowledge for mathematics teacher education. Some papers deal with what is entailed in becoming a mathematics teacher educator, by self-reports and reflective accounts (e.g., Tzur, 2001), and others describe special courses for providers of professional development activities for mathematics teachers (e.g., Even, 1999) or programmes enhancing growth of mathematics teacher educators through their practice (e.g., Zaslavsky & Leikin, 2004).

Teacher educators can be seen both as learners and as facilitators of learning. The "content" related to their learning involves beliefs, knowledge, and practice as well as some meta-cognitive skills, such as exhibiting awareness and employing reflection. A major concern of providers of professional development activities is the need to foster teachers' reflective practice. Theories of reflective practice follow Dewey's emphasis on the reflective activity of both the teacher and the student as a means for advancing their thinking (Dewey, 1933; Schön, 1983, 1987). The notions of reflection on-action and reflection in-action have been acknowledged as significant components contributing to the development of teachers as well as teacher educators' knowledge and practice. The importance of reflecting on one's own practice and learning experiences is expressed by Lerman (2001): "Reflective practice offers a view of how teachers act in the classroom as informed, concerned professionals and how they continue to learn about teaching and about learning,

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about themselves as teachers, and about their pupils as learners" (p. 39). Thus, teachers and teacher educators can be seen as constant learners who should continuously reflect on their work and make sense of their experiences. From a social-practice theory, which stems from Vygotsky's theory on the social nature of the learning process (Vygotsky, 1978), mathematics teachers and mathematics teacher educators are often regarded as two interrelated communities of practice enhancing each other's development.

A key issue to be addressed in professional development of mathematics educators is the learning of mathematics. Cooney (1994, 2001) discusses two central constructs for mathematics educators—mathematical power and pedagogical power, which deal with teachers' abilities to draw on the knowledge needed to solve problems in context (mathematical or pedagogical). Jaworski (2001) adds a third construct—educative power, which characterizes the roles that teacher educators may play in the process of enhancing teachers' learning. This construct can be taken to include the ability of teacher educators to draw on knowledge that is needed for facilitating teachers' mathematical and pedagogical problem solving.

Various models have been suggested by scholars attempting to describe teacher practices as well as teacher learning. In this chapter I describe models that have been useful to me in my own work. A model that provides a lens through which to examine mathematics educators' practices is suggested by Jaworski (1992, 1994) in her Teaching Triad, which is consistent with constructivist perspectives of learning and teaching. Her triad includes three elements, which are often inseparable: the management of student learning, sensitivity to students, and the mathematical challenge. According to Jaworski, "this triad forms a powerful tool for making sense of the practice of teaching mathematics" (1992, p. 8). By substituting "students" with "learners", and "mathematical challenge" with "challenging tasks", a more general triad is obtained (Zaslavsky & Peled, 2007) that may be applied to other learners, for example, mathematics teachers, in the context of their learning.

Jaworski's Teaching Triad can be used not only for making sense of classroom practices, but also for highlighting the different kinds of knowledge teachers need for teaching mathematics, which concur to a large extent with some of Shulman's categories (1986).

Steinbring (1998) offers a model that provides insight to the mechanisms that facilitate learning of both students and teachers in the course of a mathematics lesson. His model looks at the learning of students and their teacher as two autonomous systems that build on each other. In this model, reflection plays a critical role in both student and teacher learning. While students learn by engaging in a task, interpreting and making sense of their solutions, and reflecting on and generalizing them, the teacher learns from observing the processes students encounter, varying the learning offers, and reflecting upon the entire process. Similar to the modification of Jaworski's model, by substituting in Steinbring's model "students" with "learners" and "teacher" with "facilitator", this mechanism can be useful for making sense of how various mathematics educators learn from their practice, including mathematics teachers, mathematics teacher educators, and mathematics teacher educator educators. As shown in **Fig. 1.3.1.2**, facilitators' learning occurs as an

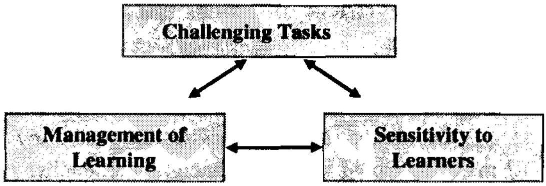

Fig. 1.3.1.1 Modification of Jaworski's Teaching Triad

outcome of their observations of learners' engagements in tasks and by reflecting on learners' work.

Interestingly, both models (**Figs. 1.3.1.1 and 1.3.1.2**) can be applied to various kinds of learning settings, differing in the specific facilitator-learner identities (i.e., mathematics teacher–students, mathematics teacher educator–mathematics teachers, mathematics teacher educator educator–mathematics teacher educators) and in the nature and content of the learning. In both models, tasks play a critical role.

Zaslavsky & Leikin (2004) suggest a recursive look at these two models that conveys similarities between the different mathematics educators yet also suggests that the type and complexity of tasks changes considerably for the different kinds of mathematics educators (mathematics teachers, mathematics teacher educators, and mathematics teacher educator educators). Thus, the knowledge and practice required of mathematics teacher educators include engaging teachers in tasks that address all three elements of Jaworski's teaching triad while exhibiting sensitivity to teachers as learners and managing the entire activities.

Similarly, a recursive look at **Fig. 1.3.1.2** (as in Zaslavsky & Peled, 2007) conveys the mechanism of learning through practice and at the same time illustrates the

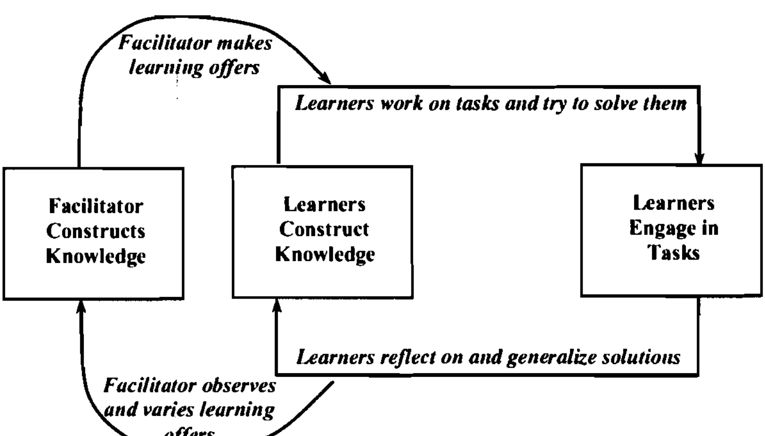

Fig. 1.3.1.2 Modification of Steinbring's Model of Teaching and Learning Mathematics (based on *Zaslavsky & Peled, 2007)*

increasing complexity of the cycles of reflections that are required of the various kinds of mathematics educators.

As mentioned earlier, these models have been useful in thinking about mathematics educators' knowledge and development (Zaslavsky, Chapman, & Leikin, 2003; Zaslavsky & Leikin, 2004; Peled & Hershkovitz, 2004). They are particularly helpful in trying to make sense of how one becomes a teacher educator. Unlike the many kinds of institutionalized (pre-service and in-service) teacher education programmes, there are hardly any teacher educator education programmes; thus, becoming a mathematics teacher educator occurs over time, through ongoing reflection on one's own experiences in facilitating teachers' learning (e.g., Tzur, 2001). Teacher educators' responsibilities usually involve both teaching and research (Adler, Ball, Krainer, Lin, & Novotna, 2005; Zaslavsky, 2007). Mostly, their research contributes to their own professional development.

The above models convey the recognition that teacher learning is highly influenced by tasks in which teachers engage. Increasing attention is given to the nature of tasks that promote teacher learning and to task design (e.g., Arbaugh & Brown, 2005). This trend is presented in detail in Zaslavsky (2003), in which they stress the importance of the type of mathematical tasks, which teachers are offered by their educators in the course of professional-development programmes.

2. An Example of a Task Stemming from a Teacher Educator's Research

Zodik & Zaslavsky (2007) present the following task, which emerged from their study on mathematics teachers' choice and treatments of examples. Consider the following problem (the Rhombus Problem):

> A rhombus $\square BDEF$ is inscribed in a triangle ΔABC. Its diagonal BE is perpendicular to the side of the triangle, i.e., $BE \perp AC$.
> Prove that ΔABC is an isosceles triangle.

In practice, such a problem is usually presented with an accompanying diagram. In Zodik and Zaslavsky's study, the authors report on a classroom observation in which the teacher shared with one of the researchers her dilemmas regarding the kind of diagram to provide. The teacher raised some subtle concerns that increased the researchers' awareness to this problematic aspect of learning and teaching (Mason, 1998). Namely, each diagram has some advantages and some limitations. This classroom event seemed to the researchers a basis for creating a genuine learning opportunity for other practising as well as prospective teachers.

Thus, the task for both prospective and practising teachers, designed by the teacher educators based on their research findings, was as follows:

Consider the diagrams–a, b, c, d in **Fig. 1.3.1.3**–which one would you choose to accompany the problem?

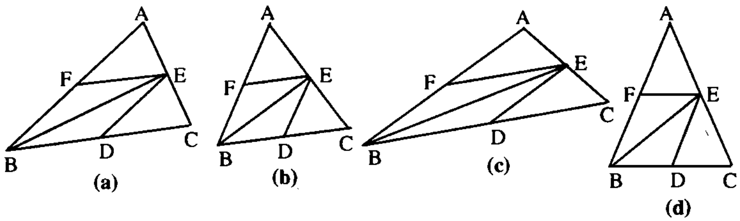

Fig. 1.3.1.3 Possible diagrams accompanying the Rhombus Problem (based on Zodik & Zaslavsky, 2007)

Diagrams a, b, c, and d in **Fig. 1.3.1.3** depict possible examples representing the givens of the problem. This task emerged from a classroom observation and was followed by an interview with the teacher, who expressed perplexity regarding what diagram to choose. By using this with prospective and practising teachers, we created a dilemma for them to consider, similar to the natural one the observed teacher experienced.

Note that the formulation of the problem does not indicate which pair of sides of ∇ABC are equal. Thus, it may not be clear at first sight what the specific goal of the problem is, that is, which of the following equalities should be proven: $BA = BC$, $CA = CB$, or $AB = AC$. A capable student could analyse the problem and reach the conclusion that an attempt to prove that $BA = BC$ makes more sense than the other options (e.g., for symmetrical considerations). However, most students do not approach this problem in such a way. Moreover, they tend to rely to a large extent on the accompanying diagram. If no diagram is provided, students are likely to act in a prototypical manner by sketching the problem situation, as in Diagram d and attempting to prove that $AB = AC$, since many students think of an isosceles triangle with a horizontal basis.

Diagram a is a rather accurate illustration of the given case. It conveys the two sides of the triangle that are equal ($BA = BC$), making it easier for the student to decide how to proceed. Diagram b is a special case of the given in which ∇ABC appears to be an equilateral triangle. Thus, it conceals the direct outcome of the given (i.e., that $BA = BC$) and may leave the student helpless regarding where to focus and what to prove. Diagrams c and d are distortions of the possible cases, as they are both "impossible": in Diagram c, ∇ABC seems like a "generic" triangle, not an isosceles one (each side is of different length). This can be perceived as a "general case" that does not disclose any hints regarding which two sides are equal. Diagram d can be seen as a misleading sketch that contradicts the given and may lead the student to an attempt to prove that $AB = AC$, which in fact cannot be inferred from the given.

As seen previously, the specific sketch accompanying the geometric problem may influence the way a student approaches the problem and the extent to which the student is successful in proving it. One may argue that the more accurate the figure the better. However, a counter argument could be that by disclosing the full picture (as in Diagram a), the task for the student changes. He or she no longer needs to

analyse the situation and make an educated choice what to prove. For a student who relies on the visual appearance, it becomes straightforward that s/he should focus on proving that $BA = BC$.

3. Concluding Remarks

Teachers' deliberations surrounding the Rhombus Problem drew their attention to subtleties they were not aware of before, including ambiguities they had transmitted to their students with respect to the role of diagrams in geometry. They had different views on what the best diagram would be and drew on their experiences and knowledge of students to support their views. The arguments they brought were both mathematical and pedagogical. This task illustrates how teacher educators' research may influence their teaching.

For mathematics educators, tasks serve a dual purpose (Zaslavsky, 2005, 2007). On the one hand, tasks are the means and content by which learning is facilitated. On the other hand, through a reflective process of designing, implementing, and modifying tasks, they turn into a means of the facilitator's (e.g., teacher educator's) learning. Moreover, in many cases research informs and enhances teaching and vice versa through tasks.

There is a fruitful interplay between teacher educator's roles as researchers, facilitators of teacher learning, and designers of tasks for teacher education. This interplay is a driving force for effective professional development of all mathematics educators—mathematics teachers, teacher educators, and educators of teacher educators.

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Chapter 1.3.2
Becoming a Teacher Educator: Perspectives from the United Kingdom and the United States

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1. Background

In countries such as England and the United States, where there remain long-standing difficulties in ensuring that sufficient numbers of well-qualified new mathematics teachers enter the profession, there are similar difficulties in the supply of well-qualified mathematics teacher educators in universities. For example, a recent national inquiry into mathematics teaching in England noted with alarm that "respondents to the Inquiry…expressed anxieties about the future capacity and availability of suitably qualified mathematics educators in higher education to deliver quality ITT (initial teacher training) and provide ongoing CPD (continuing professional development)" (Smith, 2004, para 2.29). Similarly, in the United States 43 percent of university-level positions in mathematics education went unfilled in 2006. Of the positions that were filled, half were filled by new doctoral graduates and half by people already in the field who had changed institutions (which, of course, simply created a new opening) (Reys, 2006). The statistics for 2007 are similar, although there were 129 open positions in 2007 as compared to only 86 in 2006.

The problem of the quality and supply of well-qualified mathematics teacher educators in universities is being recognised, albeit only recently, by the English Teacher Training Agency (TTA), the government body responsible for the recruitment and training of teachers. In 2003 the TTA awarded grants to professional-subject associations to produce materials and provide induction support for tutors new to initial teacher education (ITE). For mathematics, this task is being undertaken by a small group of people from four United Kingdom associations: the Association of Mathematics Education Teachers, the Association of Teachers of Mathematics, the British Society for Research into the Learning of Mathematics, and the Mathematical Association. One major outcome of the project is a website developed to support new (and existing) ITE tutors (http://www.itemaths.org.uk). This support extends across all ITE subjects and other areas of common concern: citizenship, behaviour for learning, diversity, and inclusion (multi-verse) and can be accessed through the Teacher Training Resource Bank (http://www.ttrb.ac.uk/).

In the United States, the issue of the shortage of mathematics teacher educators is being addressed with federal funding aimed in two directions. The first direction was to fund large centers (USD ten million over five years) to increase the capacity

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and infrastructure for producing mathematics teachers. The second, more recent, direction is to seed small amounts of money for small cohorts of doctoral students to work on a particular research problem in mathematics education—an approach akin to the typical approach to doctoral education in the sciences in the United States.

2. Who Becomes a Teacher Educator

The vast majority of people who become teacher educators in England are successful teachers looking for a change. Their experience ranges from being a head teacher, a head of a department, or second in a department, to having no management experience at all. Others become teacher educators after having worked as Local Authority consultants providing professional development courses and advice to teachers in school. Others have worked as freelance consultants. Several have worked with student teachers either by mentoring them during their school placements and/or running occasional sessions at the university. Many find themselves taken on as part-time tutors and continue to combine a range of work once they start work in university.

Teacher education in England has changed dramatically over the last twenty years, as government education policy has become more concerned with, and prescriptive about, the professional preparation of teachers (Furlong, Barton, Miles, Whiting & Whitty, 2000). All ITE provisions are now regularly inspected by the Office for Standards in Education using standards produced by the TTA as a yardstick (DfES & TTA, 2002).

At one time, entering higher education as an ITE tutor was considered a positive career move, as the salaries were higher and there would be opportunities to pursue "research and scholarly activity". Whilst there is still the expectation that ITE tutors undertake such activity, the salaries of school teachers have improved considerably in recent years, outpacing those of university staff, and now the prospect of moving into higher education does not have the same financial appeal. This makes it more difficult to recruit well-qualified and experienced colleagues from school to work as ITE tutors. Nationally, many institutions find they may have to advertise more than once and may fail to make a suitable appointment at all. The introduction of "Golden Hellos" for tutors new to higher education, which involves substantial payments during their first three years in post, is indicative of a problem that is not unique to education (HEFCE, 2003).

Alongside the relative decline in salaries in the higher-education sector, there has been a reduction in the amount of funding to higher-education institutions, even as student numbers have increased substantially. This has led to tutors having to do "more with less"; a recent survey of higher education institutions in England established that ITE is underfunded by around twenty percent (DfES, 2004). Whitehead, Menter, and Stainton, (1996) describe problems that full-time tutors are beginning to have as more part-time staff are used to teach ITE students and visit them in school, whilst full-time tutors take on managerial and bureaucratic roles. It is not unusual for new tutors to be employed on a part-time basis when a full-time tutor leaves.

The route to becoming a teacher educator in the United States varies greatly. Initial teacher education occurs at a variety of post-secondary institutions, including two-year community colleges, smaller liberal arts colleges, state universities, and large research universities. Initial teacher education is conducted both by mathematicians (most of whom have no formal preparation for becoming mathematics educators) and faculty members who began their own careers as classroom teachers. Such individuals may or may not hold a doctorate in mathematics education and may or may not have any formal training in mathematics education. Some are classroom teachers who are looking for a change of pace; others are graduate students employed on assistantship to teach classes. Many are adjuncts (often retired teachers) who teach part-time. The cadre of individuals responsible for the supervision of field experiences is even less well defined than those teaching in the college classroom. Still others are full-time faculty members, but even their preparation for their role differs. While there is growing recognition that teaching mathematics (and therefore future teachers of mathematics) requires a specialized knowledge base (Viadero, 2004), institutions in the United States are just beginning to grapple with what this means.

The United States suffers from a similar salary problem to England. In primary and secondary education, teachers work on a "salary scale" whereby their salaries increase with their years of service and the number of professional degrees beyond the bachelors degree. These salaries are generally adjusted on a cost-of-living basis annually with little to no merit pay. Salaries in higher education, however, are set at a base level and do not take into account years of teaching experience prior to earning the doctorate—whether in primary, secondary, or tertiary education. Annual raises are generally based on merit with minimal (or no) cost-of-living adjustment. The salaries of faculty members at research universities are generally disproportionately based on their research productivity (e.g., grant dollars acquired, publications, presentations, doctoral students graduated). Thus, it is not unusual for a school teacher to make more money than a university professor with the same number of years of experience.

3. What the Role Involves

In England, most tutors who have been involved in the induction events work on secondary Post-Graduate Certificate in Education courses, but many also teach undergraduate courses, primary courses, courses for teachers of seven- to fourteen-year-olds, and courses for teaching assistants. They also find themselves teaching masters- level continuing professional development courses for practising teachers. For those without research in their backgrounds, they are invariably expected to embark on academic study and research with a view to gaining a masters or Ph.D.

ITE tutors most enjoy working with their students. Most get a lot of support from their university colleagues. Many tutors enjoy the flexibility that working as an academic offers—managing their own diary and time to pursue academic interests.

The competing demands of tutoring students and developing research and scholarly activity is often challenging for new tutors.

Tutors quickly develop new skills, including skills appropriate to working with adult learners and skills in personal use of information and communications technology (ICT). They improve at keeping up to date with education initiatives and reading around mathematics education issues. They find themselves facing a steep learning curve as they adjust to working in a higher-education institution and adapting to the practices of assessment and quality assurance in that institution.

New tutors are often frustrated, feeling unable to do the job as well as they would like because of insufficient time. Some feel torn between teaching and research, while others find the bureaucracy and external expectations frustrating.

In the United States, the role of teacher educators generally involves teaching mathematics content courses, teaching mathematics methods courses, supervising field experiences, and/or supervising student teaching. At smaller institutions, mathematics teacher educators may also find themselves teaching courses on human development, special education, technology in education, or more general methods classes for all majors or mathematics content courses, such as pre-calculus, for all majors. At some institutions faculty members specialize in primary education, middle-level education, or secondary education, while at other institutions teacher educators span the range of kindergarten to twelfth grade.

The continuing professional education of teachers is handled either through graduate programs (leading to masters degrees) at universities or through professional development programs. Professional development programs may be run by school districts, government agencies, private non-profit agencies, or for-profit companies. The professional development opportunities range from well-organized, coherent programs to one-shot workshops on specific topics. In most states it is mandatory for teachers to earn a specified number of "professional learning units" for attending professional development courses or workshops in order to maintain their teaching license. These courses/workshops are generally taught by current or former classroom teachers. There is little to no regulation of professional development programs in the United States (either their content or who teaches them).

4. A National Programme of Support

In England, in order to support those new to the role of ITE tutor, the TTA awarded contracts to produce support materials and run induction events in most secondary subjects in 2003. There was relatively little prescription about what was to be produced. Mathematics was unique in that four professional-subject associations collaborated to produce the bid and are now responsible for ensuring that materials are generated and made available over the life of the project. Whilst this was initially for two years it has now been extended until 2009.

One of the main outcomes of the project is a website (http://www.itemaths.org.uk) developed to support new (and existing) ITE tutors. It contains links to other sites

of potential interest and support, information about discussion lists, and an archive of articles. Many of the articles have been especially written for the website by experienced ITE tutors and are designed to share approaches to working with beginning teachers, interview and selection procedures, bibliographies, and approaches to working with teachers who take a major responsibility for the beginning teacher's development whilst they are in school, and much more.

The role of ITE tutor is a demanding one (Sinkinson, 1996). The principal route into secondary mathematics teaching in England is a thirty-six-week Post-Graduate Certificate in Education (PGCE) course, of which twenty-four weeks are spent in school; for primary, teachers are trained as generalists with just eighteen weeks in school. When not teaching beginning teachers, ITE tutors are invariably teaching on other courses as well as visiting PGCE students in their placement schools. The induction events which have been run as part of the project have focused on what it means to be an ITE tutor and aspects of the role, including working with beginning teachers and working with colleagues in school (see Pope, Haggarty, & Jones, 2003). The induction events have been run at the beginning of the academic year (i.e., in September) in the hope that tutors can attend before their courses start, which is not always the case. Since 2003 approximately twenty tutors have attended these annual events. As well as the opportunity to work on what their new role involves, tutors value meeting with other new tutors and finding out about different courses across the country.

Feedback from each course has been used to inform the development of these events. In particular, the time for tutor-tutor discussion was increased, and more time was given to considering in more detail the different ways of working with beginning teachers and assessing their progress.

New tutors to ITE value the support that the project has made available to them. Other tutors are using the website, as they find it a useful gateway to mathematics education websites. Many tutors believe that the induction event is particularly valuable in providing an opportunity to think about their wider role and some of the fundamental issues, as well as sharing pragmatic concerns about ways of working.

In the United States, the National Science Foundation, an agency of the federal government, funded nine centres at USD ten million each to improve the infrastructure for producing teacher educators in mathematics in view of the retirement of the post-World War II generation known as the "baby boomers". It is too early to know the results of this investment. One of the centers, the Center for Proficiency in Teaching Mathematics, located jointly at the University of Georgia and the University of Michigan, did tackle the problem of the preparation of teacher educators (see cptm.us for examples of the work of this centre). The first funding cycle is currently under way for a new program, which will provide support for small cohorts (about five students) to conduct research on a particular problem in mathematics education as part of a doctoral program. This grant is intended to test out the typical method of preparing doctoral students in the sciences in the field of mathematics education. As the first funds will be awarded in 2007, there are no results of these efforts yet.

In 1997 a new professional organization formed in the United States. This organization is called the Association of Mathematics Teacher Educators, and it was formed in recognition of the fact that mathematics teacher education is a specialized task that warrants a professional organization with its own conference. Similarly, in 1998, the first issue of the *Journal of Mathematics Teacher Education* was published by Kluwer, again in recognition of the specialized task of mathematics teacher education and lack of appropriate venues for publishing work in this area.

5. Conclusion

The examples cited previously raise concern about the future of mathematics teacher education in both England and the United States. Who becomes a teacher educator, the motivations and incentives for doing so, and the tasks and duties of teacher educators are all worthy of further discussion on an international level. It appears certain that the recruitment and retention of mathematics teacher educators will need to be a major focus of institutions of higher education in future years.

The specialised knowledge that is needed to become a teacher educator and how one acquires this knowledge are also worthy of both discussion and research. Transition from the primary or secondary mathematics classroom to a role as a teacher educator presumably requires some specialised training and preparation, but we know little about what that training might entail. A systematic study of the support mechanisms in England might provide insight into the role that professional organizations and electronic resources might play in educating teacher educators. However, there are many countries in the world that do not have specialized professional organizations for mathematics teacher educators and where access to electronic resources is challenging. Thus, the discussion we have begun in this paper, inspired by the 15th study conference, needs to be continued and expanded to include a diverse collection of countries.

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Chapter 1.3.3
Educators Reflecting on (Researching) Their Own Practice

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This section of the book highlights the education of the mathematics teacher educator in several ways. In this chapter, the focus is on the education of the educator through his or her involvement in reflecting on or researching his or her own practice. Two ways are considered in addressing this focus: educators as reflective practitioners and educators as researchers.

1. Educators as Reflective Practitioners

Mathematics teacher educators' reflection on their own practice can be viewed as a basis of their learning in the context of educators as reflective practitioners. The works of Dewey (1933) and Schon (1987) promote the importance of reflection or thoughtful actions by practitioners. Dewey defines such reflective action as that which involves active, persistent, and careful consideration of any belief or practice in light of the reasons that support it and the further consequences to which it leads. Schon stresses the importance of reflective practitioners framing and reframing problems in light of information gained from the setting in which they work. According to Schon, reflection-in- and on-action are the mechanisms reflective practitioners use that permit them to continually develop and learn from their experiences. From these perspectives, the process of understanding and improving one's own teaching must start from reflection on one's own experience.

In the context of a reflective practitioner, then, the educators' reflection on their teaching is an inherent aspect of their work and also allows for their development and growth. Such reflection includes the educator: examining, framing, and attempting to solve the dilemmas of classroom practice; and being aware of and questioning the assumptions and values he or she brings to teaching. Thus, viewing educators as reflective practitioners assumes that they pose and solve problems related to their educational practice on an ongoing basis. Through this reflective process, they could develop new patterns of thinking with which to approach the complexities of teaching teachers. The development of reflective patterns could enable them to step back from their routine ways and consider alternative instructional choices and the impact those choices might be expected to have on their students' learning. They could

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learn about what ideas students have developed about a topic, how they understand or misunderstand the material being taught, and how different strategies work with specific groups of students—all of which can be used to guide future planning and instruction.

2. Educators as Researchers

In considering educators as researchers, the focus is now on the formal and systematical study of their teaching or the students they teach. Teacher educators can use their own teaching or students as a basis for understanding pre-service teachers' knowledge and learning and the teaching approaches they use in order to effectively facilitate pre-service teachers' learning. Such studies could be grouped into at least two broad categories:

1. Those in which the educators study their students' characteristics, for example, their mathematical and pedagogical knowledge, attitude, and classroom behavior during field experiences, in order to understand the pre-service teachers, but not explicitly their own teaching.
2. Those in which the educators study their teaching approaches, for example, a course, a program, or specific activities/tasks given to their students, in order to determine their effectiveness or the relationship between the approaches and effective or meaningful learning.

This paper deals with the second category as a basis of discussing educators researching their own practice. In particular, only studies in which the educator is involved in teaching the participants as a regular part of their teacher education programs and all of the students in the educator's class are included in the analysis, that is, not studies in which one or two students are considered. This constraint seems necessary to consider the studies to be about the educator's teaching and not merely particular students. Based on this category of research, a review of related research literature for the last ten years suggests that there are few studies that deal with mathematics educators conducting research on their own teaching. In fact, the studies reported at the 15th study conference offered nothing to this sub-theme of the book. However, of the studies identified as relevant, they provided insights into two key questions associated with the educators' learning: What did they research and how did they research it? What did they learn or could learn from these studies?

3. What is Being Researched?

The sample of studies reviewed involved pre-service teachers enrolled mainly in mathematics education courses taught by the researchers. These studies indicate that educators researching their teaching focused on investigating ways of facilitating pre-service teachers' development of mathematics knowledge and, to a lesser

extent, their instructional knowledge. Each study involved a different instructional approach. **Table 1.3.3.1** summarizes these components for a sample of these studies.

Table 1.3.3.1 Studies of Educators' Practice

Mathematics concept/procedure	Instructional Approach	Educator/researcher
Ratio and proportion	Four-component model	Ilany, Keret, & Ben-Chaim, 2004
Statistical investigation	Two investigation tasks	Heaton & Mickelson, 2002
Integer addition and subtraction	Instructional explanations	Kinach, 2002
Arithmetic operations	Investigating arithmetic word problems	Chapman, 2004
Change	Investigations with technology	Bowers & Doerr, 2001
Problem solving	Reflection and inquiry tasks	Chapman, 2005
	Two courses centered on problem-solving experiences	Roddick, Becker, & Pence, 2000
Pedagogical knowledge		
Discourse	Mathematical discourse	Blanton, 2002
Pedagogical problem solving	Problem-based learning	Taplin & Chan, 2001

For these self-studies of practice, with the central goals of understanding and guiding practice, qualitative methods seem to be more appealing in terms of dealing with small samples and exploring in depth what was happening in the courses and how it can be improved. While the details of the research process used in these studies varied, the common structure consisted of involvement in a teaching situation, some form of records of the situation, making sense of the records, and making meaningful conclusions for future use.

4. What is Learned?

Educators researching their practice are likely to learn much more about it than tends to be reported in the research literature. For the sample of studies identified previously, what the educators learned was connected to their evaluation of their teaching approaches in terms of whether they were effective in facilitating the pre-service teachers' learning. Most of these studies reported findings that indicated the instructors' teaching approaches were effective. Examples of these findings follow.

This first set of studies deals with facilitating pre-service teachers' learning of specific mathematics concepts. Ilany et al. (2004) found that the four-component model they developed for teaching pre-service teachers ratio and proportion topics was successful in producing changes in the pre-service teachers' understanding

of ratio and proportion. Kinach (2002) found that engaging secondary pre-service teachers in instructional explanations of integer addition and subtraction tasks was effective in deepening their knowledge of these concepts. Chapman (2004) found that engaging pre-service elementary teachers in analyzing and representing arithmetic word problems in a variety of ways resulted in more depth in their understanding of the arithmetic operations and word problems. Bowers & Doerr (2001) found that engaging pre-service secondary teachers in activities that introduced perturbations in their knowledge of the mathematics of change and the use of technology to assist in resolving them led to their development of a deeper understanding of the underlying quantities represented in velocity and position graphs, a more meaningful interpretation of the mean value theorem, and the importance of appropriate contexts. Finally, Heaton & Mickelson (2002) found that, for their approach to help pre-service elementary teachers develop knowledge about statistical investigations, some of the pre-service teachers mentioned learning statistical content and process but showed little progress on the more ambitious aims of the unit. For example, formulating a question that can be addressed quantitatively was problematic for them.

This second set of studies deals with facilitating pre-service teachers' learning about mathematical or pedagogical problem solving. Chapman (2005) found that engaging pre-service secondary teachers in self-reflection and inquiry activities involving problem solving resulted in their understanding and development of more realistic models of genuine problem solving. Roddick, Becker, & Pence, (2000) found that providing pre-service secondary teachers with rich and varied problem-solving experiences resulted in the participants falling on a continuum ranging from not much discernible implementation of problem solving to substantial integration of it in their teaching. Taplin & Chan (2001) found that using problem-based learning to develop pre-service primary-school mathematics teachers' skills and understanding of themselves as pedagogical problem solvers helped the participants to maintain or improve their attitude towards problem-based learning. Finally, Blanton (2002) found that using classroom discourse in an undergraduate mathematics course to challenge pre-service secondary mathematics teachers' notions about mathematical discourse resulted in their transition towards an image of discourse as an active process to build mathematical understanding and development of their ability to participate in such discourse.

In addition to the effectiveness of the instructional approaches studied, some authors offered general guidelines resulting from their investigations of their practice. For example, Taplin & Chan (2001) suggest that their approach of problem-based learning can be an effective way of facilitating teachers' development, provided that the tasks have classroom relevance and applicability; the teachers have some early experience of success to build their confidence; there is plenty of opportunity for collegial discussions; and support is given when they experience negative emotions in their attempts to implement new ideas. Blanton (2002) suggests that the undergraduate mathematics classroom (as opposed to the methods classroom) offers a powerful and unique forum in which pre-service secondary teachers can practice, articulate, and collectively reflect on reform-minded ways of teaching.

In general, this sample of studies gives some indication of what educators can learn from researching their own practice and what others can learn from their experiences in terms of understanding how to effectively and efficiently use specific classroom techniques/approaches. Such studies can expose differences between instructors' intentions and actions and what students experience or how they perceive instructors' intentions. They can provide insights of how to model a reflective approach to teaching for the pre-service teachers.

5. Conclusion

Writing about educators' reflection on, or research of, their own practice is difficult because this deals with their learning for understanding and enactment, which, for the most part, is likely to be and remain as a personal endeavor with little presence in the research literature. However, these activities (reflecting/researching) are important means of mathematics educators' education and, perhaps, deserve research to understand, for example, the questions and problems the educators pose, the perspectives they use to interpret and improve their practice, and how the processes shape and restructure their personal knowledge about teacher education and their practice. For the studies discussed in this chapter, there was no clear indication of how the educators' thinking changed and subsequent actions taken by them in relation to their practice.

The process of reflecting on, or researching, one's own practice is dependent on the context, the perspective and richness of repertoire that one brings to that context, and one's ability to draw on a level of reflection appropriate to that context. It also depends on one's ability to notice, "to be awake to possibilities, to be sensitive to the situation and to respond appropriately" (Mason, 2002, p. 7). Hence, the nature of the description resulting from these studies is necessarily incomplete and occasionally fragmented. Caution is required when attempting to generalize across settings in studies of this nature or using them as a basis of other educators' learning. On the other hand, because of educators' direct involvement in the classroom, they can bring a perspective to understanding the complexities of teacher education that cannot be matched by external researchers, no matter what methods of study they employ. They can offer to other educators information of how others really think about the situations studied and meaningful examples for other educators to use to stimulate their own reflections on their own practice.

To conclude, as a basis of their education, mathematics teacher educators' reflection on their practice can take place in the context of being a reflective practitioner or a researcher. Both of these contexts involve reflection as an integral aspect of the processes of studying one's own practice. Reflection is often initiated when the individual educator encounters some problematic aspect of practice and attempts to make sense of it. It is a process of reviewing an experience of practice in order to describe, analyze, evaluate, and inform learning about practice. It enables the practitioner to assess, understand, and learn through his/her experiences. It is a personal

process that usually results in some change for the individual in his/her perspective of a situation or creates new learning for him/her. This process of reflection, if then related into practice, can assist the individual in gaining the required knowledge, leading to a potential improvement in the quality of the learning opportunities provided to students. However, reflecting on one's own practice can be empowering in terms of accomplishing growth in one's practice but also constraining in terms of being bounded by one's taken-for-granted perspectives. The latter situation can be minimized if reflection is viewed also as a collective activity. Without the medium of relationships, reflection can lack the genuine discourse necessary for thoughtful and in-depth changes in behavior.

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Chapter 1.3.4
Educators and the Teacher Training Context

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1. Introduction

There is increasing recognition in the United States of the need for mathematics departments and mathematicians to become involved in training mathematics teachers for primary and secondary schools. Certainly, issues that are valued by both mathematicians and mathematics educators could promote collaboration. For example, the Mathematical Sciences Research Institute, in Berkeley, California, has initiated a series of workshops entitled "Critical Issues in Mathematics Education," which aims "to provide opportunities for mathematicians to cooperate with experts from other communities on the improvement of mathematics teaching and learning" (Thames, 2006, p. iii; see also, for example, Conference Board on Mathematical Sciences, 2001; McCallum, 2003). However, this area of the 15th study conference received no papers.

In this article, we discuss ideas that might be of interest to both mathematicians and mathematics educators, and we give examples of some projects that could represent ways to begin collaborations. The ideas that we discuss include the concept of the Knowledge Quartet (Rowland et al. 2005a, 2005b), the notion of a mathematical habit of mind, and the comparison of ways in which mathematicians present fractions to future teachers (McCrory, 2006). Finally, we look at preliminary results from a small international survey to consider whether and how mathematicians and mathematics educators might collaborate on content and pedagogy courses for intending teachers. It is important to find common grounds for discussion between mathematicians and mathematics educators on topics valued by both groups: when the communication between the two groups goes astray, difficulties such as the so-called "Math Wars" in the United States (Ralston, 2003) can result in the waste of much valuable energy.

2. Ideas of Mutual Interest

In the past, in many countries, the relationship between mathematics departments and teacher educators has been minimal, to say the least, especially at research-

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oriented universities. However, there is research concerning training of primary teachers that has proved to be of much interest to the mathematics community. For example, Liping Ma's work (1999) is well known and resonates well in the community of mathematicians regardless of whether they are interested in teacher training, because it shows why mathematics content is important for intending teachers.

Similarly, a recent project in the United Kingdom, "Subject Knowledge in Mathematics", explored how mathematics subject knowledge of trainee elementary school teachers influences their classroom teaching performance (see Rowland, Huckstep, & Thwaites, 2005a,b). Based on extensive analysis and coding of videotaped lessons taught by trainee teachers, the authors conceptualized a theoretical framework that describes certain aspects of trainee teachers' actions in the classroom and how these are influenced by their content knowledge. The framework is called the Knowledge Quartet and consists of foundation, transformation, connection, and contingency. Detailed description of the findings can be found, for example, in Rowland et al. (2005a). Further use of this framework has shown that it is a comprehensive tool for thinking about the ways in which subject knowledge comes into play in the classroom. In several publications by the Cambridge team (see, for example, Rowland et al., 2005b), there are examples of how the Quartet has been used to analyze the classroom actions of trainee teachers. Data-grounded research such as this is of potential interest to mathematicians, as it highlights how mathematics content knowledge relates significantly to the ways trainee teachers act in the classroom.

Equally influential, Hyman Bass's (2005) interpretation of mathematics education in the spirit of a field of applied mathematics has given credence to the importance of teacher preparation as a substantive field for application of mathematical study. McCallum's 2003 article adds depth to the conversation through the discussion of the similarities and differences between the nature of research in mathematics education and other mathematics disciplines. There are now some institutes (e.g., the Institute for Mathematics and Education and the Park City Mathematics Institute) that provide a variety of ways for doctoral-level mathematicians to become active in primary or secondary education.

In a review of some textbooks authored by mathematicians and meant for future teachers (McCrory, 2006), a key point is the comparing and contrasting of specific approaches to fractions from a variety of conceptual viewpoints. Given that a rigorous approach requires equivalence classes, which no one believes is the right approach to teaching future primary teachers (nor their pupils), each of the authors constructed fractions in different manners. Fractions are thus a natural place for mathematicians to see how deep the mathematics of the primary grades is. A joint seminar discussing this article would provide a venue to explore common interests between mathematics and mathematics education, as would a seminar to discuss the work of Liping Ma or the philosophy of Hyman Bass.

3. Mathematical Habit of Mind

It is difficult to trace the history of the idea of mathematical habit of mind (MHM) because it is something that mathematicians do all the time. It somehow becomes

part of the internal thinking process. For example, it is present in all of Polya's writing on problem solving although he doesn't use the phrase. A fine example is Polya's 2007 article, which encourages students to reason "through analogy and verification in special cases" and to exercise "plausible reasoning" (inductive reasoning in more modern terms). The discussion of the area of a triangle in terms of the length of its sides is highly informative and delightful.

The Conference Board on Mathematical Sciences (CBMS) has had a strong influence on the courses taught in the United States by mathematicians to future teachers.

> Two general themes of this report are: (i) the intellectual substance in school mathematics; and (ii) the special nature of the mathematical knowledge needed for teaching. We owe to mathematics education research of the past decade, or so, the realizations that substantial mathematical understanding is needed even to teach whole number arithmetic well (CBMS, 2001, p. xi).

The notion of the mathematical habit of mind is mentioned explicitly in one of the CBMS recommendations:

> Recommendation 4: Along with building mathematical knowledge, mathematics courses for prospective teachers should develop the habits of the mind of a mathematical thinker and demonstrate flexible, interactive styles of teaching. ...*Teachers need to learn to ask good mathematical questions, as well as find solutions, and to look at problems from multiple points of view* (p. 8, italics in original).

There are other possible descriptions of MHM. One is contained in Thames: "Throughout this document, the term knowledge is used broadly to include the knowledge, skill, dispositions, and habits of mind that support doing mathematics" (2006). Another example is contained in the preface to Long et al. and describes MHM as follows:

> One of the ways to enhance conceptual understanding is for future teachers to develop a deeper way of thinking about mathematical concepts and problem solving, to explore mathematical ideas, to formulate questions, and to ask themselves whether there is "something more" (a generalization) in the mathematics on which they are working. This trait is called a "mathematical habit of the mind" in some recent writings or, more simply, a habit of the mind (MHM). MHM includes open ended or vaguely worded problems in which you will have to examine how you would model a situation (2008).

Here are two examples that have been used in teacher preparation courses. The students in course Math for Future Primary Teachers at the University of Kentucky have found that a traffic-jam problem (Peter-Koop, 2005) is quite intriguing as a group exercise. The students were astounded by the success of the German fourth-graders who tried the problem, as reported by Peter-Koop.[1] Another example of MHM is found in a course for intending teachers of middle school pupils (children ages 12–14, grades 6–8, or 7–9 in the United States). After investigating a number of examples exploring the sum of even integers and the product of odd numbers, the instructor would ask the students to prove that the product of even numbers is even. The students were asked if "something more is going on." After some discussion, they realized that the product is divisible by four as well as two. This also is an

[1] There is a 3-km traffic jam on the motorway. How many vehicles are caught in this traffic jam?

example of a MHM that can be introduced to primary-grade students and to their teachers.

MHM should be reserved for ideas and problems in which significantly more is going on than meets the eye. On the other hand, problem solving, communicating, reasoning, and making connections, all principles of the National Council of Teachers of Mathematics, can also be described as habits of mind.

The previous argument advocates the inclusion of MHM in the curriculum of courses for intending teachers. To our knowledge, in spite of Recommendation 4 of CBMS as quoted previously, there is not yet a research base for this approach. Furthermore, MHM is now in the process of being constructed for a curriculum in an American high school. The articles (Cuoco, 2001) and (Cuoco, Goldenberg, & Mark, 1996) are position papers setting out a direction for the mathematics curriculum and drawing upon a curriculum-development project, Connected Geometry (for more details, see www.phschool.com/cme). However, the ideas promoted in these papers are familiar to those who have been involved in innovative curriculum developments in mathematics over the last fifty years. The thinking is implicit in Polya's books on problem solving (written by a mathematician) and in more recent publications, such as Mason, Burton, and Stacey (1985) (written by mathematicians and mathematics educators). In many countries, some of this thinking has found its way into national curricula but with limited influence. In England and Wales, the development of a strand of the mathematics national curriculum entitled "Using and Applying Mathematics" was intended to develop awareness of some of the things that Cuoco calls MHM, and the introduction of assessed coursework for public assessment at age sixteen was intended to promote this, but assessed coursework has recently been abandoned in the face of fears about widespread plagiarism. Even now, twenty two years after the introduction of the national curriculum in England, with "Using and Applying Mathematics" at its heart, many of those teaching mathematics in English schools are not confident about encouraging their pupils to develop mathematical habits of mind. The role of MHM or the development of mathematical thinking in school mathematics is fertile ground for joint work between mathematicians and mathematics educators but, given space considerations, we only have dealt with, and advocated, MHM in the curriculum for future teachers.

4. Content/Pedagogy Course Survey

We now report briefly on some preliminary findings emerging from the content/pedagogy survey. In the survey, which consisted of an online questionnaire, we asked mathematicians and mathematics educators for their views about the roles of content and pedagogy courses in the context of initial teacher training, about the actual and desired role of mathematics educators in the content courses, and the actual and desired role of mathematicians in the pedagogy courses. We received thirty-four responses from fourteen countries; twenty of the respondents described themselves as mathematics educators, four as mathematicians, and ten as both. We

used the institutional frame of the University of Kentucky (where the first author works) as an example of a possible structuring of teacher training, and we asked respondents to comment. The first emergent finding is that, in some institutions (four in our sample), there is no division between content courses and pedagogy courses. Mathematics content is taught together with its pedagogical content. Other respondents, from institutions where this is not the case, also expressed a preference for this way of structuring the course. One respondent commented:

> It is not very healthy to separate content courses from methods courses. Content courses (for future or practicing teachers) should have a strong methods component (or at least a component that links the content to research in mathematics education); and methods courses should have a very strong content component.

For the role of content courses, there is agreement among the respondents that future teachers need to have the "big picture" of mathematics and that they should be able link the mathematics that they will eventually teach to other parts of the subject. This would seem to relate to the finding of Askew, Brown, Rhodes, Wiliam, and Johnson (1997), in a study of effective primary mathematics teachers, that effective teaching involves a connectionist approach to mathematics. The affective dimension is also mentioned in the responses; the case is proposed that trainee teachers (especially at primary level) often come to content courses with their own "mathematics anxiety", which needs to be addressed if they are to teach mathematics confidently and not transmit their own anxiety to their students. As for pedagogy courses, respondents suggest that such courses should aspire to teach theoretical constructs in mathematics education and how these bear on teaching practice. However, the findings most relevant to our chapter emerged from the final questions.

4a. The Actual and Desired Role of Mathematics Educators in Content Courses

Across the responses there is an almost equal split between institutions where mathematics educators teach content courses and where mathematics educators have no input on such courses. There are however a few institutions where such courses are taught jointly by mathematicians and mathematics educators. As for what mathematics educators can bring to these courses, there is overwhelming agreement that this is "knowledge of mathematics for teaching". This is articulated in several ways ranging from the application of teaching and learning theories in problem-solving activities to knowledge of new development in mathematics education.

4b. The Actual and Desired Role of Mathematicians in Pedagogy Courses

The most common response is that mathematicians have no role in teaching pedagogy courses. As for the desired role for mathematicians in such courses, there is a

spectrum from respondents who believe strongly that mathematicians should indeed have no role in pedagogy courses (with some very strong support for this thesis) to respondents who think this role should, for example, involve mathematicians sharing their own experience as researchers and learners, offering the "big picture" of mathematics beyond the curriculum taught in content courses and offering epistemological ideas on what is mathematics. One respondent wrote:

> The best thing would be that mathematicians (I intend: mathematicians with no competence in mathematics education) do not teach those courses! Otherwise it would be like if they would teach specific medical disciplines to prospective medical doctors, based on their experiences of ill people and auto-diagnosis and auto-therapy...

Another wrote:

> I think mathematicians and mathematics educators should collaborate in planning and teaching both the content and pedagogy courses. Mathematics education is an interdisciplinary field and both courses benefit from the different perspectives and expertise of both mathematicians and mathematics educators.

In conclusion, the main findings of this small survey point to the following:

1. A split between pedagogy courses and content courses might not be productive. It is possible to teach courses that address both at the same time.
2. The main role of mathematicians in a content course is perhaps the ability to offer to student teachers the "big picture" of mathematics and develop students' epistemology of mathematics. This will help students form their pedagogy.
3. The role of mathematics educators in content courses is to provide theoretical frameworks within which the teaching of mathematics can be conceptualized.
4. There is no agreement on what the role of mathematicians in pedagogy courses should be. There is some strong support for the thesis that mathematicians will not be able to contribute to pedagogy courses, as pedagogy is not their area of expertise. On the other hand, many responses pointed to the opportunity to capitalize on mathematicians' knowledge of mathematics and "how mathematics works" and their experiences as learners and teachers of mathematics.

In considering these points, it is important to recognize that nearly all of the participants considered themselves to be either mathematics educators or both mathematician and mathematics educator. In particular, points 3 and 4, stated previously, might suggest a positive perception of mathematics educators and a rather critical image of mathematicians among respondents. There is another interesting issue connected to the results of our survey that comes from the first question, when the respondents were asked to describe themselves as mathematicians, mathematics educators, or both. In the survey we omitted a clear definition of what qualifies someone to be a mathematician or a mathematics educator. Eight of our respondents felt the need to describe their academic qualifications and the type of work they do in response to this question, for example, indicating that what makes someone a mathematician can be open to discussion. An investigation into the decision processes used by respondents to determine whether they considered themselves as

mathematicians, mathematics educators, or both might shed light into why, for example, mathematicians are viewed rather critically by the participants in the survey.

Our small survey suggests that there is much common ground as well as some significantly different views between mathematicians and maths educators. We hope that this short paper might encourage more dialogue between the different communities.

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Initial Mathematics Teacher Education: Comments and Reflections

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During a recent government inquiry into the quality of school education (Senate Standing Committee on Employment, Workplace Relations and Education, 2007),

> the committee received more evidence in relation to the quality of mathematics teaching than on any other aspect of the curriculum. Many of the submissions and much of the testimony was critical to the point of being pessimistic about the likelihood of improved standards, as well as being fearful of a further decline in standards and performance (p. 58).

It is unlikely that the problems and deficiencies identified in the Australian education system are confined to that country alone. A section devoted to initial mathematics teacher education in a volume that captures the deliberations and practises of thoughtful mathematics educators brought together by an international study on the professional education of mathematics teachers is thus timely indeed.

As expected, the section contains a smorgasbord of information, practices, and trends. In the bulk of the contributions, the contents rely heavily on information supplied by those who attended the special study, although in those chapters reference is also made to sources from the field more broadly.

In the opening chapter, Maria Teresa Tatto and Stephen Lerman search for patterns within and across a multitude of teacher education systems. Some level of regulation for teacher education programs is common, and teacher preparation is most frequently an undergraduate rather than graduate activity, but academic entry prerequisites, course organization, and content—including time spent learning mathematics and practicum/field experiences—all seem to vary widely. The diverse landscape sketched offers a useful context for the different programs, many linked to stages of teachers' professional lives, described in the remainder of the section.

Likening initial teacher education to a braiding process, one that enables initially discrete strands of knowledge for teaching to be fused into a coherent, unified, and integrated body of knowledge (Peter Liljedahl et al.) conjures an evocative image of a teacher education program for which ingredients, individually and collectively, are informed by best practice rather than expediency considerations and which meld relatively seamlessly into a coherent and well-rounded preparation for beginning teachers. The chapter, with its emphasis on the what of teacher education, serves as a constructive precursor to the two contributions that follow and in which aspects of the how of teacher education are explored.

R. Even, D.L. Ball (eds.), *The Professional Education and Development of Teachers of Mathematics*, DOI 10.1007/978-0-387-09601-8_13,

Uwe Gellert's overview covers a number of best "practices" rather than best "practice" examples from teacher education programs, many supplied by the 15th ICMI Study attendees. Each is deemed worthy of further consideration and possible emulation or adaptation by other practitioners. Capturing these different activities, and their rationale, not only gives voice to a larger number of the study's participants than might otherwise have been possible, but also offers useful glimpses of curriculum perspectives in different countries.

The "practicum" setting, as Christer Bergsten and his co-authors make clear, provides excellent potential for learning from, by, and in doing. For this to be achieved the field work needs not merely to be well integrated into the rest of the program but also to be planned and organized purposefully. Examples are provided of different theoretical perspectives to guide the development of programs and of shorter term and specifically focused activities. Their common aim is to expose aspiring teachers to activities and settings which will foster their independence and enhance their mathematics teaching expertise.

In each of the next three chapters, voice is again given to multiple participants. The attempts to recognize the input of so many participants is both laudable and challenging. Contributions concerned with attitudes and beliefs about aspects of mathematics teaching and learning, whether relating to student teachers or their instructors, are collated by Stephen Lerman. Merrilyn Goos focuses on different dimensions of pre-service teachers' school experiences and particularly on the transition from learner to practitioner. The realities faced by beginning teachers are drawn together by Carl Winsløw.

The themes highlighted by Lerman and his team are certainly familiar: the challenge to broaden the mathematical horizons of those disenfranchised during their earlier studies, ensuring that when more positive attitudes and beliefs about mathematics are achieved these are indeed reflected in instructional practices used in the classroom and the tension between the time needed to learn mathematics and exploring how it might best be taught. Advice for overcoming dysfunctional beliefs about mathematics and its teaching are mostly couched in general terms in the excerpts included in the chapter; thus, specific strategies must be inferred.

The journey from student to teacher is rarely smooth. Ways to facilitate the passage are described by Goos and by Winsløw in their compilations of the contributions that deal with this transition. Explanations of hurdles encountered and proposals for better routes to be explored reflect the contributors' different interests—itemized as pre-service courses, school environment, students' own histories, knowledge, beliefs and attitudes in the first of the two chapters, and as transition at an epistemological, an institutional, or a personal level in the second chapter. A range of theoretical stances inform and are informed by the different emphases. The different transitional pathways—some informally sanctioned and others formally endorsed—covered in Goos's and Winsløw's summaries highlight the many obstacles to be overcome. Through the voices of the study participants the reader gets useful insights into problems which appear universal and those which are seemingly exacerbated or minimized by contextual or system specific factors.

For the concluding chapters the focus changes to teacher educators, that is, from those who survive the voyage from trainee to certified mathematics teacher to those in charge of the extensive journey. For this the editors have gone beyond the contents covered at the 15th ICMI Study to capture a variety of international perspectives.

Orit Zaslavsky reminds us that becoming a mathematics teacher educator is a lonely task. There are few formally structured entries into the profession, and opportunities for professional development are largely self-initiated. Close scrutiny of the responses to carefully devised, and possibly reworked, tasks can—it is argued—be particularly instructive for mathematics teacher educators wishing to improve their own mathematics learning and that of their students.

Sue Pope and Denise Mewborn retain the lens on teacher educators in their discussion of the who, how, and what in English and American contexts. The historically limited formal preparations into the profession, the gradual whittling away of attractive conditions, and the burden of increasingly diverse and demanding duties are balanced by reports of the—still limited—introduction of induction and support programs. There is no shortage of proposals for action for those wishing to improve the preparation and continuing education of mathematics educators.

The theme of teacher educators learning by doing and reflecting, introduced with reference to tasks in Zaslavsky's contribution, is expanded by Olive Chapman. Her review of mathematics teacher educators learning from interrogations of their own practices indicates that such explorations typically focus on facilitating the development of pre-service teachers' mathematics knowledge rather than on the promotion of instructional knowledge. A too-ready acceptance of "one's taken-for-granted perspectives" points to the limitations of relying on teacher education through self-studies despite the undeniable benefits to be reaped from such work.

Chapman's review reiterates the imbalance, noted in earlier chapters, between projects and attention directed at content versus pedagogy. The final chapter, with its emphasis on collaborations between colleagues in university mathematics and education departments serves as a welcome antidote. The actual and putative collaborative activities described deserve wide dissemination. Early responses to a survey questioning the roles of content and pedagogy courses in pre-service programs and the ideal input by educators into the former and mathematicians into the latter elicited a wide range of responses—some productive, and others harshly destructive. New structures, it appears, will not be welcomed universally.

The care taken in most of the chapters to give a clear voice to those who contributed to the 15th ICMI Study through discussions during, and written input prior to, the study is both liberating and restrictive. The chapters are undoubtedly enriched by the multiple, diverse, and international dimensions introduced. At the same time their scope is influenced, guided, and in parts bounded by the work prepared for the 15th ICMI Study. Nevertheless, core aims of the study have been met: to foreground the work of teachers, to compare different systems and traditions, and to identify and disseminate successful and productive initiatives, including those previously hidden because of language barriers.

Where do we go from here? At the beginning of this commentary I referred to a recent government inquiry into education. Space restrictions prevent all the

recommendations to be listed, but many remarkably echo the views promoted in this section. For example, "restructuring teacher training courses so as to encourage and require aspiring secondary teachers to commence their studies in. . .relevant disciplines before undertaking specific studies in education by degree or diploma"; ". . .schools and school systems (should) take particular measures to improve teacher professional development in mathematics"; and "the government takes steps to improve remuneration of teachers so as to raise the profession's entry standards and retention rates by providing incentives" are all themes rehearsed as well in the previous chapters.

What comfort can we take from this convergence? Can we assume that the combined and congruent views expressed by international researchers and a national, government-funded inquiry will necessarily lead to constructive action? Or should we recognize that actions beyond the dissemination of volumes such as the one spawned by the 15th ICMI Study are needed to translate the rhetoric of governments and of groups of international researchers into programs beneficial to teachers, their teachers and their students? Highlighting the urgent need for insightfully planned, balanced, well-founded, and adequately funded programs for teacher training without detracting or minimizing from the already sterling work being done is an international and ongoing challenge.

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Initial Mathematics Teacher Education: Comments and Reflections

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Mathematics teacher preparation is always a formidable task in practical mathematics teacher education and is a significant concern in mathematics education in general. Thus, the section entitled "Initial Mathematics Teacher Education" will be attractive to many scholars in this area.

From the introduction, "Overview of teacher education systems across the world", readers obtain a clear and comprehensive overview of current systems in use by different countries and school districts around the world and are also exposed to the different features of a variety of systems as well as the explicit and implied ideas and beliefs behind them. The longer and more detailed descriptions and comments provide us with a broad background and comprehensive information that will assist us in better understanding those chapters with special themes.

In the chapters under the topic "Student teachers' experiences and early years of teaching", the authors report research on university pre-service teachers, student teachers, and new mathematics teachers; the relationship between university learning and "real-live" teaching; and the relationship between the theoretical and practical aspects of teaching and learning. Such a relationship constitutes a vital link in the chain of initial pre-service training sessions for mathematics teachers. Thus, the theoretical and practical combined analysis, for example, of the Zone of Proximal Development, the Zone of Free Movement, and the Zone of Promoted Action (Goos, this volume) is very important, and I hope it will serve as an exemplar of similar research.

The section entitled "Mathematics educators' activities and knowledge" especially caught my eye. From this we learn that at the preliminary stage, people usually pay more attention to the external aim they expect to achieve. It is only when they have successfully gone farther that they wish to improve themselves by pursuing higher and newer objectives. I optimistically view the theme on teacher educators' development as representing the progression of research on mathematics teacher education. I mainly comment on chapters in this section, but some views may apply to the chapters under "Student teachers' experiences and early years of teaching". As Zaslavsky and Peled (2007) suggest, there is much in common with

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the knowledge and development of mathematics teachers and that of their educators, which is conceptualized as parallelisms by theme editor Pedro Gómez.

One of the key and urgent tasks with which we are confronted is how to better equip mathematics teacher educators. Thus, the profound and substantive research area covered by the chapters under "Mathematics educators' activities and knowledge" is worth exploring. I hope these papers can assist in forming a new agenda for the mathematics education community.

As theme editor Pedro Gómez points out, it is actually in the primary stage of the development: "There are few such studies". However, chapter authors have handled some key elements in which a central point of view—reflection—is emphasized. It is really a practical and theoretical concept in the process of teacher educators' growth. For example, "unlike the many kinds of institutionalized (pre-service and in-service) teacher education programs, there are hardly any teacher-educator education programs, thus, becoming a mathematics teacher-educator occurs over time, through ongoing reflecting on one's own experiences in facilitating teachers' learning" (Tzur, 2001). In other words, the number of available teachers to teach us is small. However, in the context of a reflective practitioner, then, the educators' reflection on their teaching is an inherent aspect of their work, which also allows for their development and growth (Chapman, this volume). This may imply that, at least in an early stage, we have to rely on this way to enhance our internal mathematics power and pedagogical power to meet the needs of new external tasks of teacher education. Thus, self-teaching may be the only approach in which reflection is pivotal.

There is a Chinese proverb: "Teachers open the door. You enter by yourself". It means that a teacher can only play the role of a guide. However, real progress comes from personal struggle. People should be encouraged to learn from own experiences actively and autonomously. Individual reflection (and that of teacher educators as a whole) could stimulate them to master a higher level of knowledge and skills.

On the other hand, self-reflection is not enough. The mathematics education community should also try to find more ways for "collective reflection". Special symposiums, conference sessions, a special column in journals, special issues of journals, personal visits, and personal communication can play such roles. For example, a mathematics education association can edit a series of books for the benefit of teacher educators.

The outcome of reflection is always at the heart of valid self-reflection. In this volume, Zaslavsky reviews the literature and compares and analyzes several models and then shows an example of a mathematics task of a triangle with a rhombus. This example provides mathematics teacher educators with a way to reflect.

The conclusion of the chapter is illuminative: there is a fruitful interplay between teacher educators' roles. "We regard this interplay as a driving force for effective professional development of all mathematics educators that take part in the instructional hierarchy—mathematics teachers, teacher educators, and educators of teacher educators".

Further Reading

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Section 2
Learning in and from Practice

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1. Background

"Learning in and from Practice" is the title of the second of the major strands in the conference on the Professional Education and Development of Teachers of Mathematics. Fundamentally, this strand considers the learning of pre-service and experienced teachers as they engage in the practice of teaching mathematics. The Discussion Document sets out the thinking and rationale behind this section of our study; the collection of papers accepted for the strand provides a range of approaches to studying teachers' learning and highlights both important particularities and general complexity.

Before presenting an overview of the chapters in this section, it seems relevant to speak briefly about the ways in which the content of our strand has been addressed and the nature of the work that has produced these chapters. Our work at the conference was organised into four groups. Accepted papers were assigned to just one of the groups, and a group consisted largely of the authors of these papers. At the stage of formation of groups it was difficult to identify a particular theme for a group due to the great diversity of papers.

The leading team for the strand consisted of Chris Breen, Barbara Jaworski, Marja van den Heuvel-Panhuizen, and Terry Wood. Regretfully, Joaõ Felipe Matos, who had done much of the preparatory work for the strand, was unable to attend the conference. Each of us took on the role of coordinator for one group. We read and organised our papers to suggest a programme of work for the group sessions. We had five sessions of work that were highly participative with consideration of all the papers through critical reviews. In the fifth session we came together as a strand, each group presented their work from the first four sessions, and finally we sought some synthesis from our deliberations. The synthesis which emerged formed the basis of planning for this study volume.

Four themes were identified and agreed by the editorial team as the basis for chapters; editors of the four chapters, in the order they appear, are Barbara Jaworski, Joaõ Felipe Matos, Marja van den Heuvel-Panhuizen, and Terry Wood. The agreed themes follow.

Theme 1—Development of teaching in and from practice (Barbara Jaworski, editor): What is known about characteristics of the process of developing professional

expertise in the teaching of mathematics? What factors are recognized? What issues are we aware of? Are there benchmarks in teacher development?

Theme 2—Processes of learning in and from practice (Joaõ Felipe Matos, editor): What is the role of reflection (cognition), collaboration (social learning), communication (development of shared repertoire and meanings in pedagogy), and social institutional or socio-cultural environment?

Theme 3—Models, tools, and strategies to support learning in and from practice (Marja van den Heuvel-Panhuizen, editor): What models, tools, and strategies exist that can be used for teacher learning across school-age-level settings and cultural institutions? This theme deals with issues such as instructional tasks, video cases, lesson studies, sharing and reflecting on learning experiences, communities of learning teachers, and e-learning.

Theme 4—Balance of mathematical content and pedagogy (Terry Wood, editor): What is the relationship between the development of mathematical content and pedagogical development?

Since it had been agreed in the IPC that the writing of chapters should follow the participative model created by the conference, all participants in the strand were invited to become authors and all who responded to the invitation were included in the writing process. We suggested the chapter to which each person might contribute and simultaneously allocated the original papers to one of the chapters. Thus, each chapter has a team of authors, a set of papers, and an editor. It was up to the chapter team to decide how to address the substance and material of their theme. It was agreed that:

- reference should be made to all papers relevant to the theme of the chapter.
- the contributory authors would not necessarily all have a major role in the writing, although all those who wished could read and comment on what was written and their names would appear with those of the main authors.
- material from the conference papers might be included directly in the chapter as *boxed* items with author name attached. This has been done in Chapters 1 and 3, Themes 2.1 and 2.3 below.

The four chapters in this section, as you will see, have broadly followed the themes, but of course writing has been strongly related to the interests and experience of the authors. Both conference organisation and production of this study volume have aimed for diversity and inclusion so that the outcomes might represent experience and scholarship as widely as possible. Thus the chapters focus on national particularities, as well as presenting a scholarly perspective on the thematic areas.

2. The Four Chapters/Themes

The four chapters/themes are both distinct and deeply inter-related. Chapter 1, Theme 2.1, sets the scene by considering overt *distinctness* and *inter-relatedness*. Through consideration of factors, benchmarks, and issues, the chapter both categorises elements of teachers' learning in practice and emphasises its complexity. Categorisation must not be reductionist—a significant tension for scholarship in this field. In order to make sense of the learning of teachers and development of teaching, we need

to find ways of identifying key elements of learning in/from practice; however, we must maintain with integrity the complexly nuanced relationships that are embodied in the teaching process. The chapter weaves examples from the papers that were the basis of study to point to specific factors, benchmarks, and issues while continually coming back to the global complexity of the substance of our analysis.

Chapter 2, Theme 2.2, addresses the processes of teachers' learning in and from practice, and suggests that a social theory of learning can underpin our observations and analyses. Starting from Sfard's use of acquisition and participation metaphors to dichotomise epistemological frames, the chapter zooms in on *social practice theory* as one way of viewing learning in practice. Drawing on the work of Lave and Wenger, it offers an analysis of *community* and *practice*, and the concept of *community of practice*, to situate teachers' learning both for the individual in the group and for the group as a whole. It takes examples from contributions to the conference and uses their particularities to emphasise ways in which these theoretical ideas can illuminate developmental practices that result in teachers' learning and teaching development.

Chapter 3, Theme 2.3, offers a striking complementarity to Chapter 2. Whereas Chapter 2 foregrounds the social theory, exemplified through practice, Chapter 3 brings the practice to the foreground, taking social theory as its backdrop. In Chapter 3, we are taken into a deep consideration of the roles of tools and settings for which communities of practice are a fundamental theoretical basis. Thus the chapter presents and analyses models for teaching development, such as lesson study, video cases, and e-learning, drawing on particular research emphases in different parts of the world. Transformation of practice through developmental activity in practice is the essence of the chapter, which ends with just a hint of the importance of these models for the learning of teacher educators as well as for the teachers to whom they are directed.

All three chapters so far consider *knowledge of teachers* and *knowledge in teaching*, but their consideration is largely implicit. In Chapter 4, Theme 4.4, we see an overt focus on knowledge and the nature of knowledge. The chapter starts from a recognition of the distinctness and inter-relatedness of *content* and *pedagogical knowledge*, emphasising complexity. It takes Kilpatrick, Swafford, and Findell's five-strand basis for students' mathematical proficiency (conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition) and uses this as a basis for considering teachers' knowledge. The chapter focuses on five studies from different nations to contextualise distinctions and characterise complexity. This final chapter is complementary to Chapter 1 through its analyses of knowledge in practice that identify distinctions while not losing sight of the complexity of the settings.

3. Learners in an from Practice

As we deal with concepts and strategies distilled from research analyses, it is important to keep in sight the teaching task, the students for whom teaching is designed, the subject, mathematics, and the cultural context in which teaching takes place. The

complexity of inter-relationships has been emphasised to differing degrees through the four chapters. What is it that we learn from these analyses that enables the practice of teaching mathematics to students in schools and classrooms to develop and become more effective? In asking, "What is it that we learn...", and juxtaposing this question with our topic of teachers' learning in and from practice there seems to be an important parallel between the focus on teachers' learning and this focus on our *own* learning in theorising from research analyses. The words "we" and "our own" refer to the mathematics educator researcher who is engaged in making sense of the teaching enterprise and of working with teachers to promote teaching development.

The parallel can be seen at a number of levels. Perhaps the most obvious is that, when considering tools and tasks (Chapter 3), we consider simultaneously tasks for use with students in learning mathematics and tasks for teachers, designed perhaps to enable teachers to expand their understanding of how they can use tasks in the classroom. Research provides insights at one or both levels and informs our practice as teacher educators working with teachers. We draw attention to teachers' epistemological frames, backgrounds, and positionings (Chapter 1) and learn from research on teachers' beliefs and attitudes. Simultaneously our own epistemological frames expand. For example, the chapters show that we analyse models of professional development and teacher learning within both Piagetian and sociocultural frames. In our study of practice, whether the practice of learning mathematics in the classroom (e.g., considering children's strategies and errors) or the practice of working with teachers to consider students learning (e.g., looking with teachers at classroom video to analyse students' conceptions in algebra), theoretical frames provide focusing devices to enable us to make sense of some part of the wider complexity. Regardless of whether explicit in our practice (as educator-researchers), this understanding of the relationship between the field of practice and theories that support a study of practice informs the ways in which we work with teachers and creates environments for teachers' learning.

Thus, an important parallel in considering teachers' learning in and from practice is our own learning in and from practice and how these two cohere. Chapters 2–4 all include examples from research into practice. For example, we see the practice of lesson study (Chapters 2 and 3) or the practice of using classroom artefacts to help teachers gain new ways to think about their students' algebraic understanding (Chapter 4). These practices are those of teacher educators working with teachers to promote development of teaching. Research analyses how teachers respond to such developmental practices, both in the developmental setting and in the related classroom setting. The problematic relationship between the two settings—how enhanced thinking in the developmental setting influences the classroom setting—has been a source of question and analysis for some years (e.g., Jaworski & Wood, 1999; Fennema et al., 1996). Lesson study brings the two settings together powerfully so that the developmental setting includes the classroom setting. Other related models build on the lesson study idea, drawing in a theoretical component alongside the study of practice, for example, in learning study (Marton & Tsui, 2004), design research models (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003), and in developmental research (Gravemeijer, 1994; Jaworski, 2006).

In the 15th ICMI Study we are focusing on the professional development of teachers of mathematics. These four chapters certainly provide insights into teachers' learning in and from practice. However, as we read the chapters, appreciate, and make sense of their categorisations and particularities, of their theoretical frames and relationships with practice, struggling with the broader complexity, it is *we* who are learning. A question for all of us as we read and worked on the content of these chapters concerned how this learning informs our practice and in what ways this does or can relate to teachers' practice. Stigler and Hiebert (1999) make a case forcefully for focusing on teaching development rather than teacher development. If teaching is to develop, then two groups of professionals need to be involved, the teachers and the educators. If both are involved in researching the practices and issues of teaching and development, then the parallels can be brought together. Perhaps this is why lesson study and other related models seem currently so powerful.

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Theme 2.1
Development of Teaching in and from Practice

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1. Overview

This chapter is principally concerned with the characteristics of the process of developing professional expertise in the teaching of mathematics in and from practice. The chapter comprises three sections, in which we discuss, respectively,

- Section A: Recognized Factors in Teacher Development,
- Section B: Recognized Benchmarks in Teacher Development, and
- Section C: Recognized Issues in Teacher Development.

In the discussion of "factors", we review the beliefs, experiences, and structures that have been identified as significant in studies and programs concerned with the development of mathematics teachers and teaching. This discussion frames the second section of the chapter, in which we delineate some of the markers or "benchmarks" of effective teaching practice, focusing in particular on how these have been used by researchers to study teacher development and effectiveness. Finally, in the "issues" section, we review some of the structures—conceptual, institutional, cultural, and so on—that enable and constrain research into and efforts to support teacher development.

Further to the established literature, we draw both on papers that were reviewed and accepted as contributions to the study conference and on notes distilled from our discussions during the conference. When the writing from a paper informs this writing, the relevant quotation from the paper is displayed in a box and attributed to the author. A list of authors, e-addresses and titles of the papers can be found at the end of the chapter.

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2. Section A: Recognized Factors in Teacher Development

Perhaps not surprisingly, the sorts of factors that tend to be identified as significant in discussions and studies of mathematics teacher development are focused on teachers themselves, including, for example, their beliefs about learning, their experiences with mathematics, and their attitudes towards formal education. This emphasis echoes the tendency among educationists to focus on the individual as the "site" of learning and knowing, although there are clear indications that the focus is being elaborated upon to include more collectivist interpretations of learning and knowledge. As such, although the first three of our four categories of "recognized factors" are concerned specifically with teachers, it should be understood that we are not identifying teachers as being positioned as either the problems or the solutions nor are they free of responsibility in the contexts in which they work. Our position is that what happens depends on the teachers but is not determined by them. Rather, the three categories of teacher beliefs, backgrounds, and positionings are identified as phenomena that must be interpreted and addressed, simultaneously, at personal, social, and cultural levels. This point is re-emphasized in our fourth category, in which structures for intervention are discussed.

2a. Factor 1: Teachers' Epistemological Frames

Theories of knowing and knowledge have been among the most prominent issues in the mathematics education research literature over the past several decades. We have witnessed a proliferation of perspectives that, on the surface, seem to have little in common other than an explicit rejection of the cause-effect mechanisms, representationist assumptions, and rigid mental/physical (inner/outer) distinctions that are typical of behaviorist and cognitivist frames that prevailed through much of the twentieth century.

However, close examinations of constructivist, constructionist, socio-cultural, and emergentist theories reveal some important intersections, including a shared metaphoric commitment to "bodies" (e.g., physical bodies, bodies of knowledge, student bodies, the body politic), an explicit embrace of evolutionary dynamics, and a concern with adaptive coherence rather than absolute correspondence (Sierpinska & Lerman, 1996). Embraces of such notions appear to constitute an important factor in teacher development (see related boxes, Kidd and Cedillo & Santillàn), whether applied at the level of the individual, the social/collective, or some other level. These theories entail significant shifts from traditional emphases in mathematics classes.

For example, theories that focus principally on personal understanding (e.g., constructivism and socio-constructivism) posit the child as a coherence-seeking sense-maker, requiring the teacher to structure tasks that simultaneously interrupt current understandings while providing the tools needed to modify those understandings or to construe new, more viable interpretations (Steffe & Gale, 1995).

> The implication for teachers' pedagogical practices is that tasks need to challenge students to mentally organize new information into their existing knowledge. This construction of knowledge implies that knowledge is not static but dynamic and can be manipulated and moulded (Kidd, 2005, p. 2).

> [S]ocio-constructivist theories demand that the teachers change deeply their knowledge on learning and teaching mathematics.... [S]ocial constructivism assumes, among other premises, that each student gets to the classroom with his own ideas and that the teacher must provide him with new experiences that induce him to collect data to affirm them or to refute them (Cedillo & Santillàn, 2005, p. 1).

Theories of learning that are more concerned with interpersonal dynamics and social and cultural contexts offer compatible but different recommendations for teaching. More concerned with the patterns of interaction, relational dynamics, and group expectations of social bodies, these theories see knowledge as inextricably linked to social positioning and context-specific action (Lerman, 2000). Cognition in this frame is always a collective phenomenon: embedded in, enabled by, and constrained through the social phenomenon of language; caught up in layers of history and tradition; and confined by well-established boundaries of acceptability (see related boxes, Jaworski and Hodgen).

> Knowing and knowledge are *both* attributes and building blocks of the community *and* outcomes of reflective activity of individuals within the community. We have to account for development of practice both for *individual* teachers and educators and for the *communities* to which they belong. ...[W]e can see knowledge as both situated and distributed within a community (Jaworski, 2005, p. 2).

> [C]hange and learning are facilitated by a "combination of engagement and imagination" which enables identification "with an enterprise as well as...view[ing] it in context, with the eyes of an outsider. Imagination enables us to adopt other perspectives across boundaries and time...and to explore possible futures...[and] trigger new interpretations" (Hodgen, 2005 [citing Wenger, 1998, p. 217], pp. 2–3).

This manner of theorizing can be further elaborated by extending the notion of "learner" to encompass not just individuals and social collectives, but any self-maintaining, adaptive, coherence-seeking group or phenomenon (see related boxes, Brown and Davis & Simmt).

> [T]he teacher or the teacher educator learns about their students' strategies and behaviours in learning, as a learning culture develops in the group. Such learning cultures can also develop within a group of teachers working together in a school where the head of department is metacommenting on ways in which they want the group to work (Brown, 2005, p. 3).

> ["Learning" is used here to refer to] the co-specifying activities of agents whose collective activities exceeds the summed capacities of agents. . .includ[ing] neural activity, individual understanding, classroom collectives, the body of mathematical knowledge, and society (Davis & Simmt, 2006, p. 1).

Clearly, the range of perspectives encompassed by this "factor" in teacher development is broad. However, to reiterate, what appears to be critical is not what or who is identified as the specific site of learning/knowing but the fitness-oriented, coherence-seeking, self-maintaining, context-dependent dynamics that are understood to be at work in moments of learning/knowing.

2b. Factor 2: Teachers' Backgrounds

The topic of teachers' backgrounds is often separated into the major categories of professional preparation and teaching experience. The former of these is typically further subdivided into such categories as pre-service education, in-service training, courses in mathematics, and courses in pedagogy.

Of course, there is clearly some value in these sorts of categories and subcategories. However, a consistent theme across the papers that served as the foundation for this chapter was that such fragmentation does not necessarily help to illuminate the critical factor of teachers' backgrounds, owing in part to the tendency to frame the discussion in terms of "required knowledge" and "necessary experiences".

For example, it is not difficult to find instances of effective mathematics teachers who have little formal preparation in the subject or ineffective teachers with extensive disciplinary knowledge. How, then, might one frame the topic of teachers' backgrounds in a manner that does not reduce the complexity of the phenomenon yet does not render it impossible to research and interpret?

Davis & Simmt provide at least a partial response to this question. In their study of teachers' disciplinary knowledge, they noted that a greater number of formal mathematics courses did not ensure more effective pedagogy. However, stronger backgrounds proved important when individuals were asked to deconstruct particular mathematical concepts. That is, when stronger disciplinary backgrounds were coupled with particular sorts of in-service experiences, teachers were able to generate much more profound and robust insights into specific concepts and, further, were able to suggest implications for pedagogy that were informed by their teaching

experiences but that had not previously been part of their teaching. In other words, mathematics background, in-service opportunities, and teaching experience were treated as three inextricably intertwined aspects of teachers' backgrounds.

Jaworski reaches a similar conclusion, although she articulates it in a somewhat different way. Further complicating the matter by implicating researchers in the development of teachers knowledge, Jaworski attends to such paired phenomena as knowledge and learning, insider and outsider, and individual and community (Jaworski, 2003). The underlying sensibility is similar to that articulated by Davis & Simmt: teachers' backgrounds should not be construed in terms of discrete elements but in terms of co-implicated, complementary, and mutually amplifying aspects (see related boxes, Davis & Simmt and Jaworski).

> [R]esearch focused on exclusively "questions of mathematics" or "questions of learning" is inadequate in efforts to understand teachers' mathematics-for-teaching (Davis & Simmt, 2006, p. 5).

> [L]earning is rooted in socio-cultural settings: school communities, educational communities, societal norms, project expectations, all contributed to activity and knowledge growth (Jaworski, p. 5).

Cedillo & Santillán make the same point in yet another way. Developing the metaphor of teaching algebra as a language, they point to the neurological, psychological, interpersonal, and cultural dimensions of knowing—which occur all at once. The lens provided by this metaphor prompts them to a provocative conclusion with regard to teachers' backgrounds as it pertains to teacher development. As they note, the "most resistant teachers to change were the more experienced" (p. 6) whereas "a characteristic noticed in the teachers with little experience and an insufficient subject knowledge was their good attitude towards the project" (p. 7).

On the surface, this conclusion might seem to conflict with that reported by Davis and Simmt. In one case, subject matter knowledge and teaching experience proved enabling; in the other, they were impediments. Such apparent tensions only underscore the complexity of the notion of "teachers' backgrounds", reaffirming the need to configure this construct in complex rather than reductive terms. To re-iterate, the "factor" of teachers' backgrounds comprises the disciplinary knowledge and other aspects of their formal preparation, their ongoing teacher experiences, and their opportunities for in-service work.

2c. *Factor 3: Teacher Positioning*

Mathematics teaching, of course, is about more than personal beliefs and histories. It is a radically contextualized phenomenon and must thus be considered in light of

educational cultures, professional interactions, local expectations, and so on. Further, as Hodgen develops, the relationship between context and teaching cannot be understood in unidirectional or causal terms (see related box, Hodgen).

An individual's identity, and ultimately legitimacy, within a community depends not simply on their acceptance by the community, but on the individual's identification with it (Hodgen, 2005, p. 4).

Hodgen draws on Lacanian psychoanalytic theory to introduce matters of desire, motivation, and identity to the discussion of teacher action. In Lacanian theory, pleasure is seen as dialectically linked with pain. It thus provides a way of locating the motivation to sustain change in the face of the very real difficulty of this for teachers. Lacan conceives of identity in terms of unattainable completeness. As with the aspects that contribute to teachers' backgrounds, these matters cannot be isolated and treated separately but must be considered as part of an evolving constellation of identifications that affect and are affected by the contexts of teachers' work. For example, as Hodgen elaborates, there is not just "a personal and emotional investment in professional change but also a compulsion to change" (p. 4). Hence, Hodgen concludes, "a potentially productive strategy in teacher education would be an explicit focus on such affective issues and the facilitation of the desire" (p. 5).

Also interested in matters of motivation, Brown focuses more on "purposes", which she describes in terms of ideas or aims that serve to orient efforts. Brown's "purposes" have much in common with Hodgen's "desires", contributing to student teachers "gaining a sense of the teacher they want to become" (p. 2). "Purposes" (Brown & Coles, 2000) figure into student teachers' evolving teacher identities, as articulated choices of strategies and emphases closely enough linked to this new world of the classroom that they see the need for action. They cannot be reduced to private dispositions but must be understood in terms of complex unfoldings of agent-in-context (see related box, Brown). For instance, a common problem for teachers when meeting a new group of students is, "How do I know what they know?", but experienced teachers have a range of teaching strategies that they use implicitly to find out. Seeing the need for finding out prior knowledge allows a range of strategies to be discussed and tried out by student teachers in awareness of them. Philosophical descriptions of practice do not link so easily with actions.

Purposes seemed to be in the middle position between philosophical attitudes and teaching behaviours in the classroom (Brown, 2005, p. 1).

Kidd's research into what happens to the teacher when a curriculum changes might be considered as a counterpoint to Hodgen's and Brown's discussions. She notes that the extent to which teachers embrace curriculum change varies across regions and appears to be linked to their expectations. This observed simultaneity of situational and affective influences provides support for the need to attend to matters

of desire and purpose, understood in radically contextual terms. We might ask, how are notions of expectation and desires/purposes linked or counterposed?

2d. Factor 4: Structures for Intervention

Processes, models, and tools to support learning in and from practice are the foci of other chapters in this section, and so we will limit our comments in this chapter—linking our remarks in particular to some of the emphases developed in the discussions of the preceding three factors.

In particular, and re-emphasizing the concurrency of these phenomena, a consistent theme across discussions of the structures of pre-service and in-service experiences is the need to consider structural issues (e.g., patterns of interactivity, opportunities for collaboration, blurring the roles of teachers and researchers) and conceptual matters (e.g., epistemology, disciplinary knowledge, teacher motivation) as co-implicated, mutually affective phenomena. More colloquially, one cannot separate what one knows from the structures in which one comes to know. With that point in mind, we look across some of the structures for intervention that were presented in the various research papers, underscoring how they might complement and enable the factors already identified.

Perhaps the most consistent structural feature among the research projects is some manner of joint action in which individual needs and interests were addressed in ways that supported more collective aims. This point was articulated in terms of the emergence of "inquiry communities" (Jaworski), "adaptive (or learning) systems" (Davis & Simmt), "collective participation" (Siemon), "sustained conversations" (Dawson), and "egalitarian dialogue" (Bairral & Giménez). One might characterize these collaborative moments in terms of a "collective we" rather than a "collection of me's", in which roles are blurred, intentions elaborated, and possibilities enlarged (see related boxes, Jaworski and Dawson). For instance, a community of practice becomes a community of inquiry when participants engage together in "critical alignment" (Jaworski, 2006).

> **[T]he project meetings developed a small inquiry community in which teachers came to see themselves as researchers (Jaworski, 2005, p. 5).**

> **The mentors. . .melded themselves together into functioning professional development teams, they are engaging others from their local educational community. . . (Dawson, 2005, p. 6).**

An important aspect of these moments of collectivity is that they arise around focal practices—that is, specific, focused, and sustained engagements in specific activities or with particular topics. Dawson's teams of mentors, for example, cohere

around the shared work of providing professional development opportunities to teachers in Micronesia. This project is sustained over years and involves intense five-day institutes to delve into the sorts of issues discussed previously, as well as through more extended practices, such as lesson studies. Phrased differently, these collectives are articulated not around who they are but around what they are doing.

Siemon describes a completely different approach designed for a different context and framed by different concerns, which also embodies notions of collectivity and focal practice. She comments on a "behind-the-screen" approach, by which video technologies are used to enable unobtrusive observations of classroom activity. These observations frame discussions of student learning, and group members develop strategies and vocabularies to describe practices in ways that resonate with teachers' actual experiences of teaching.

Within these sorts of collective structures and focal practices, the role of the teacher/leader appears to be configured not in terms of directing or overseeing but in more participatory ways. For example, Brown (see related box) uses the term "metacommenting" (adapted from the work of Bateson, 2000, p. 137), to describe an aspect of the role through which she foregrounds the teacher/leader's responsibilities in setting the ethos of the collective. The notion also highlights that the work and learning of the collective is dependent on the teacher/leader, but it is certainly not determined by that person.

The students are learning through [their teacher's] metacomments that "it's all right to be wrong" and that part of the culture of their classroom is "sharing different responses and methods" (Brown, 2005, p. 5).

Bairral & Giménez echo a similar sensibility. In their report, they point to a new vocabulary for describing this transformation in the role as they offer such terms as "Burning-Animator," "open provocateur," "supporter," and "team player" (these and other terms are developed in more detail in Section 3). These terms are particularly provocative when considered in the context of their study of the use of Internet space to enable dialogic learning across distances in Brazil—underscoring that the critical aspects seem to be a means and a focus for collective action.

2e. Summary Comments on Section A

In some cultures of contemporary educational research, there seems to be a strong temptation in a discussion of factors to identify the most basic and fundamental elements that constitute a phenomenon and, once identified, to consider those aspects in isolation. This tendency is a troublesome one when it comes to enterprises as complex as the development of mathematics teaching expertise. Drawing on a mathematical analogy of sorts, the factors discussed here should be understood as composite ones that are curiously irreducible to more primary elements.

Indeed, as we review the descriptions of the four composite factors described previously, we note that they cannot be considered apart from one another. The constructs of "teachers' epistemological frames", "teachers' backgrounds", and "teacher positionings" are little more than interpretive tools that entail artificial distinctions and temporary ignorances. They would likely not lend themselves to checklists and surveys but must be understood as inextricably intertwined with one another and complexly co-implicated with grander social and cultural structures.

The value of these constructs, then, lies not in the possibility for dissecting the process of becoming an effective teacher but for supporting appreciations, through taking multiple perspectives, of the contingencies of teaching and of learning to teach.

3. Section B: Recognized Benchmarks in Teacher Development

In this section, we take on the matter of what it might mean to operationalize the factors identified in the previous section—that is, to translate them into tools of observation, structures of intervention, and markers of transformation.

The section is organized into three categories of benchmarks. The first two of these pertain to teacher beliefs—or, more specifically, to the manners in which teachers are able to articulate their convictions (Benchmark 1: Explicit beliefs) and the ways that, and the extents to which, those convictions play out in their teaching practices (Benchmark 2: Enacted beliefs). These discussions are organized in parallel manners, as we delve into beliefs about the learning of mathematics, beliefs about the nature of mathematics learners, beliefs about the nature of mathematics, and beliefs about the teaching of mathematics.

Our third category, Teacher attitudes (Benchmark 3), cuts across the two other categories. In it, we discuss the sorts of dispositions that may or may not be explicit and may or may not be readily discernible in practices, but are nonetheless likely to have a profound shaping influence on "the way teachers are" with mathematics, learners, and formal education.

As in our discussions of factors, we think of benchmarks as emergent, evolving, and intertwining terms. Although we attempt to delve into aspects of these complex markers in the discussions that follow, our examinations should not be interpreted as attempts to pry these categories into subcategories.

3a. Benchmark 1: Explicit Beliefs

Given the prominence of the topic of teachers' epistemological frames in the contemporary research literature, it is not surprising that the topic of teachers' explicit beliefs about the learning of mathematics was a central theme in many of the papers considered in this strand.

For example, Brown's discussions of purposes (in reference to "the sorts of guiding principles that student teachers found energising when learning from their own experience," p. 1) and metacommenting (in effect, a means to render explicit "students' strategies and behaviours in learning," p. 3) are examples of pedagogical strategies that are intended to enable and compel prospective teachers to be explicit about what they believe about the learning of mathematics. Cedillo & Santillán echo the same sensibility. Also situating their research in the context of pre-service teacher education, they emphasize the importance of requiring "that the teacher know the state of development of mathematical thought of his students" (p. 1).

Although located differently, experienced teachers choosing to explore their own practice, Jaworski's research emphasis on "[l]earning in practice from a study of practice" is suggestive of similar emphases on finding ways to articulate what and how one learns. In this case, practising teachers work together to render explicit what is believed about learning and how those beliefs give shape to their teaching practices and research efforts. In a parallel manner, Chiocca focuses specifically on teachers examining their practices in relation to how they become aware of and work with pupils' need for correction as a means to become explicitly aware of the model of learning that informs pedagogical decisions.

Explicit beliefs about mathematics learning, of course, are tangled together with explicit beliefs about the nature of mathematics learners. Several of the reports highlighted the importance of bringing these beliefs out into the open. Cedillo & Santillán (see related box), for example, identify such articulations as an integral element in effecting teacher change.

> **[I]t is required that the teachers make evident…that they understand [their students to be] intellectually creative, able to make non-trivial questions, to solve problems and to construct theories and reasonable knowledge (Cedillo & Santillán, 2005, p. 1).**

Similarly, in Kidd's study of varied mathematics teaching strategies, the perceptions or beliefs teachers had about their students and of how they learn were indicative of their teaching practices. In particular, Kidd notes, these beliefs come to be articulated in terms of expectations of what students will and will not be able to do—which in turn contribute to decisions about classroom tasks and pedagogical approaches.

Davis and Simmt take the discussion of learners in a different direction, complexity science, which is principally concerned with phenomena that spontaneously arise in the interactions of relatively autonomous agents and that, in this spontaneous emergence, demonstrate traits and capacities that are not manifest by any of the agents on their own (Davis & Sumara, 2006). Invoking complexity science, which they define as the study of learning systems, they argue that any self-organizing, self-maintaining, and self-referencing collective can be regarded as a learner—opening the door to the possibility of treating classrooms and other social groupings as coherent learners in and of themselves. Although introducing quite different sets of entailments, the suggestion is useful here insofar as it underscores how teachers'

abilities to be explicit about their beliefs about learners can enable and orient their teaching practices.

Davis and Simmt actually extend their definition to encompass systems of knowledge, such as mathematics. This move prompts them to investigate the figurative and experiential underpinnings of mathematical concepts, casting these as fluid and interactive aspects that open new interpretive possibilities when combined. That is, they demonstrate how being explicit about beliefs on the nature of mathematics is an important benchmark in the development of teacher knowledge, given the profound influence that such beliefs can have on pedagogy. Kidd confirms this point with reference to an interview question, not cited in her paper, "What do you think of the body of mathematics as a whole?", one of the prompts used to make sense of teachers' practices and their attitudes towards change (p. 3).

Kidd's larger research focus is actually explicit beliefs about the teaching of mathematics—which comprises explicit beliefs about learners, learning, and mathematics. It is also a topic of broad shared interest among researchers. For example, both Dawson's work with novice and experienced teachers and teacher mentors and Siemon's work with groups of practising teachers are organized around the entangled issues of mathematical content, conceptions of learning, and associated pedagogies. These examples serve as important reminders that explicit beliefs should not be treated as independent strands of knowing.

3b. Benchmark 2: Enacted Beliefs

There is a tendency, within discussions of formal education, to focus on explicit, stated knowledge. However, prompted by psychoanalytic, structuralist, post-structuralist, pragmatist, enactivist, and other interpretive frames, there has been a recent surge in interest in tacit knowledge (Polanyi, 1958)—that is, in interpretations and beliefs that are so deeply inscribed in a teacher's being that they are difficult or impossible to articulate, even though they are embodied in moment-to-moment actions. Indeed, such tacit knowings might conflict with explicit statements about what is known and believed (see related box, Brown).

In the case of student teachers beginning the process of entering a new world of the classroom, without a range of effective behaviours, what seems important is that a structure for learning is put in place by the teacher-educator that supports the students in the move to implicit effective behaviours (embodied actions) (Brown, 2005, p. 3).

Of course, explicit and enacted knowledge cannot and should not be treated as distinct categories. They are, rather, dynamic complements. Explicit understandings are rooted in embodied knowings, while they orient and fade into enacted knowledge. Brown's work with pre-service teachers is organized around this notion, as she works with student teachers to render explicit otherwise tacit beliefs, focusing in particular on "purposes", such as needing to find strategies to find out what their

students know and "basic-level categories", which are linked to action and are the most simple ways in which we come to see the world or learn (Lakoff, 1987).

Unfortunately, although acknowledged as important by many, few of the papers considered in this chapter actually discuss or present strategies for tracking and interpreting enacted beliefs. For example, around the topic of enacted beliefs about the learning of mathematics, Kidd discusses the importance of setting intellectually rich tasks, Cedillo and Santillán argue that the profoundly embodied competence of using language is an apt metaphor for mathematics learning, Bairral and Giménez highlight the dialectic of explicit experimentation and implicit knowledge and beliefs, and Davis & Simmt point to the profound differences that arise when learning tasks are organized around assumptions of the vibrant sufficiency of knowledge, good enough to do what is needed (versus a belief that teaching is about correcting deficiencies). The two phrases, "vibrant sufficiency" and "static deficiency" are used to draw attention to the sorts of dynamics assumed to be operating in knowledge-producing systems. "Vibrant sufficiency" is rooted in an assumption of evolutionary dynamics, calling up notions of ongoing adaptation, mutual affect (with other systems), and coping. The measures of "truth" in this frame are adequacy and utility—that is, so long as something continues to work it is likely to persist. By contrast, theories of knowledge organised around the principle of "static deficiency" tend to assume that there is an ultimate, unchanging truth (usually thought to be located outside the knower or knowing system) that the agent must strive to attain. That is, truth is held to be static, and the knower is regarded as necessarily deficient in relation to that truth.

Similarly, on the topic of enacted beliefs about the nature of mathematics learners, Brown uses the construct of "metacommenting" to foreground the assertion that learners are embodied knowers who move back and forth between explicit and tacit actions. Hodgen's interests in motivation, identity, and desire are oriented by a similar assumption—that is, that the non-conscious dimensions of mathematics learners and mathematics learning vastly exceed the conscious dimensions.

As for enacted beliefs on the nature of mathematics knowledge, Davis and Simmt highlight some of the problems that arise when certain beliefs are allowed to remain implicit. In particular, they comment on the issues that can arise around the literalization of metaphors—that is, when teachers treat figurative aspects of images, analogies, gestures as though they were isomorphic with associated mathematical concepts and immutable. They also comment on possible pedagogical implications of enacted beliefs around the emergence of mathematical knowledge, drawing a distinction between recursive elaborative and accumulative processes. In a more pragmatic vein, Brown comments on the use of "purposes" as a useful "way of influencing [student-teachers'] developing images of mathematics...at the superordinate [philosophical] level" (p. 4).

As with explicit beliefs, implicit beliefs about learning, learners, and mathematics all seem to coalesce in enacted beliefs about the teaching of mathematics, including teachers' inclinations to attend to student sense-making (e.g., Brown; Cedillo & Santillán; Kidd), their attitudes towards errors (e.g., Chiocca; Kidd), the extent and nature of their textbook usage (Kidd), the likelihood of group work or student-led

discussions (Brown; Kidd), and the tendency to delve into the implicit structures of mathematical concepts versus treating concepts as sites for technical mastery (Davis & Simmt).

3c. Benchmark 3: Teacher Attitudes

Teachers' attitudes and predispositions are topics of emergent interest, as indicated in an *Educational Studies in Mathematics* special issue on the topic "Affect in mathematics education: Exploring theoretical frameworks", although these matters are currently difficult to locate in the matrix of established research in mathematics education. The dearth of research is likely related to the tendency to regard the individual student as the locus and fundamental unit of learning. With that assumption in place, matters of the teacher's affect or inclinations might seem relatively unimportant.

However, with the noted shift towards thinking in terms of collectivity, situated action, and contextual ethos, teacher attitude is coming to be understood as a vital element in occasioning and sustaining interest among students. Brown's remarks on teacher curiosity and Davis & Simmt's commentary on the pleasure that teachers seemed to derive in moments of joint mathematical inquiry are thus germane.

However, the most frequently mentioned topic within this benchmark is critical reflection (see related boxes, Dawson and Siemon). Bairral and Giménez comment that "Enactive and reflective processes are the basis for growing professional development" (p. 1), and Jaworski foregrounds the role of critical reflection, coupling it with inquiry as two mutually amplifying tendencies. Dawson also indicates how groups of teachers can affect another's attitude towards critical reflection.

> The ability of the project to achieve its stated purpose of nurturing effective mathematics instruction in novice and experienced teachers is dependent upon the achievement of a number of goals, including but not limited to: . . .developing novice and experienced teachers' and mentors' abilities to reflect critically on their practices and on their growth as mathematics teachers and educators (Dawson, 2005, p. 3).

> A disposition to reflect on practice and work collaboratively with peers to review and explore the teaching of mathematics is recognized as. . .crucial. . .. An emergent professional language. . .alongside other forms of peer observation and review, appear to offer powerful means of supporting and engaging teachers in ongoing, professional learning based on insightful and informed reflection (Siemon, 2005, p. 2).

Critical reflection, in other words, is broadly regarded not only as an important attitude, but also a responsibility—and thus constitutes a vital part of this particular benchmark.

3d. Summary Comments on Section B

At the risk of being too repetitive, once again we emphasize that the benchmarks presented here—that is, explicit beliefs, enacted beliefs, and attitudes—should not be treated as elements that can be isolated but as components of a more complex whole. The overarching concern here is how a mathematics teacher is present and presents (self, knowledge, purpose, etc.) in the mathematics classroom.

It is notable that the research into explicit beliefs far outweighs research into enacted beliefs and teacher attitudes. This disparity might point to a need to develop operational definitions of these constructs (i.e., of noted aspects of enacted beliefs and teacher attitudes) in order to enable observation and interpretation.

4. Section C: Recognized Issues in Teacher Development

There is a risk with framing factors and benchmarks, as we have, in terms of multi-faceted and evolving forms. It is important to observe that these sorts of complex constructs do not lend themselves to reliable and replicable observations, measurements, predictions, and manipulations.

In some ways, that is precisely the point being made. Such ideals are anchored in long-standing practices of isolating, de-contextualizing, and otherwise separating or reducing phenomena. In this section, we look across some of the ways that these tendencies manifest themselves in teaching and research practices, endeavoring to articulate attitudes that are more attentive to the evolving characters of the phenomena we study and seek to affect.

4a. Issue 1: Making and Unmaking Distinctions

Throughout the process of writing this chapter, we have been confronted with the conflicting needs, on the one hand, to use such distinctions as individual/collective, knower/knowledge, explicit/enacted, and teacher/researcher and, on the other hand, to be suspicious of these sorts of distinctions.

For the most part, we have resolved the apparent conflicts by framing these dyads as complementarities rather than dichotomies—a move that might have some important conceptual and practical advantages. For example, conceived as a dichotomy, the individual/collective dyad prompts attentions to competing interests and incompatible goals. There seems to be an implicit win/lose, good/bad judgemental position associated with the separation.

By contrast, thinking in terms of complementarities—that is, in terms of a win-win, mutually beneficial logic—prompts attentions to the manners in which individual self-interest can serve the collective good and how collective structures can enable individual growth. This sort of thinking certainly seems to underlie the

emphasis on collectivity that is so prominent in many of the research reports (as noted in the discussion of Factor 4: Structures for intervention).

Other dyads that might be similarly re-construed (and some of the reports in which they are invoked) include:

- explicit (formal) knowledge and enacted (implicit, tacit) knowledge (Brown; Davis & Simmt);
- educational researcher and teacher educator (Brown; Davis & Simmt; Jaworski; Siemon);
- researcher and teacher (Davis & Simmt; Jaworski);
- theory and practice (Brown; Jaworski);
- formal mathematics and school mathematics (Davis & Simmt); and
- insider and outsider (Dawson; Jaworski).

This said, some dyads, which appear to be read as strong distinctions, were invoked in the reports, including:

- vibrant sufficiency versus static deficiency (Davis & Simmt); and
- constructivist pedagogy versus traditional pedagogy (Kidd).

4b. Issue 2: Contextual and Situational Matters

Another possible entry to the list of troubling dyads would be action-and-context, which is a topic that has figured prominently in discussions of mathematics, education, and mathematics education informed by culturalist, sociological, and anthropological literatures.

Dawson flags some of the dimensions of context, including geography and culture, that are often allowed to slip into the backdrop of contemporary discussions. Geographical matters are rarely raised in discussions of mathematics education. By contrast, culture is much more prominently represented in the literature, but it is perhaps given short shrift in mainstream discussions. The "simple" matter that the language of expression for this volume is English—which might be argued to be co-implicated in the Eurocentric biases that are knitted through most of formal mathematics—is a case in point. Dawson's discussion presents a number of cautions, rooted in his experiences of learning to be mindful of where one is, whom one is with, and how one is with them.

Kidd's examination of some of the regional differences in mathematics pedagogy in the United States also provides a cautionary tale of sorts. The variations in attitudes, emphases, and outcomes further underscore that care must be taken to be attentive to the nuances of context.

In addition, of course, such attention must be sustained. A prominent theme across the research reports was the need to study practices, beliefs, and transformations over extended periods—not only because individuals change and collectives evolve, but also because they and their contexts/situations are enfolded in and unfold from one another. Davis and Simmt express this sensibility in their

acknowledgement that they affect what they are studying in their efforts to study phenomena associated with mathematics pedagogy—the point being that sensitivity to context entails an attunement to one's role in defining and triggering changes to context.

4c. Issue 3: Embracing Complexity

This point amounts to a re-iteration of one that we have made throughout this chapter: when seeking to understand phenomena that are contextually sensitive and dynamically specified, one must not construe "factors", "benchmarks", "issues", or any other tool to enable observation and interpretation in isolated, reductive terms. The question emerges of how to focus research on specifics while maintaining a hold on complexity.

For researchers and teacher educators, we might argue that certain ethical issues present themselves here. The following are among the issues that merit consideration:

- How do we study a phenomenon that is not only constantly changing but that changes in part because it is being studied? In particular, how do we track our own complicity in the phenomena that we study?
- How are assumptions concealed and/or exposed by our languaging practices?
- What is the role of new vocabularies in helping us to think/act differently (including, e.g., "virtual monologue", "metacommenting", "vibrant sufficiency") with regard to our roles as researchers, teacher educators, teachers, and mathematics knowers.

4d. Summary Comments on Section C

An unfortunate aspect of educational research is that, as an academic enterprise, it was first articulated in an era when inquiries were to be framed by well-articulated questions, unambiguous definitions, and pre-selected methods—and those inquiries were expected to provide correspondingly robust, replicable, and universally applicable results.

An emergent issue in research in the social sciences and humanities is that desires for validity and reliability, drawn from analytic science, have given way to desires for viability and reasonableness, rooted in more complex awarenesses. Mathematics, education, pedagogy, and research are evolving forms—ones that are entangled in, but not able to be reduced to, the actions of those who engage in or identify with them. However, there are also moves to pull back from vague qualitativity to provide clear definitive constructs and warrants for evidence (Shavelson, Phillips, Towne, & Feuer, 2003).

5. Concluding Remarks

What is being said in the research papers presented to this strand of the study group recognizes the complexity of teacher development. Knowledge is situated in the contexts of the practices of countries, within both mathematics learning and teaching practices. An individual teacher's practice and development is difficult to talk about in isolation without considering the local nature of the practices involved at different focal lengths of the lens viewing those practices ranging from governmental, school, particular class culture, and achievements of students. To be able to consider working with teachers who are themselves learning about the teaching and learning of mathematics working together in groups across schools (Jaworski) or across islands of Micronesia (Dawson) seems like an important construct. Co-observation and co-teaching—being able to act in different ways through observing a different reality, planning together, or developing a culture within a department or within a classroom that itself becomes a learning community or classroom (Davis & Simmt, Brown) gives us a way of thinking about professional development in which the teachers share thoughts and practices rather than a particular way of doing things. Teachers learn, and those who teach teachers learn correspondingly.

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Theme 2.2
Mathematics Teachers' Professional Development: Processes of Learning in and from Practice

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1. Overview

In this chapter we claim that many of the changes and current approaches to professional development reflect not only educational changes but also a significant change in our understanding of what constitutes learning. New approaches to learning lead to new organization of professional development initiatives for teachers, and the move from an acquisition to a participation metaphor brings changes to the ways in which the professional development of mathematics teachers is conceived and implemented. In contrast to the training model of professional development, we characterize the practice-based model as focused primarily on expanding teachers' participation in the practices of what constitutes teaching. We contend that a move from the training model to the practice-based model for the professional development of mathematics teachers can be understood as a consequence of the move from the acquisition to the participation metaphor for learning.

2. Introduction

2.1. The Training Model of Professional Development

Less than fifty years ago, when mathematics teachers engaged in professional development activities, it was often the case that these teachers would take a mathematics content course at a nearby university, usually during the summer. This model of professional development flourished, for example, during the New Math Era, when there was a tacit understanding that teachers lacked the mathematical content knowledge needed to appropriately teach school mathematics (Freundenthal, 1978)

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and that the new mathematical ideas "were completely foreign to most mathematics programs and teachers in 1960" (Fey, 1978, p. 341). In these professional development initiatives, teachers, as individuals, engaged in activities that aimed at increasing their own knowledge of mathematics, either by looking more closely at particular topics or by learning about a new mathematical development. Teachers who sought such kind of professional development often came back to the classroom reinvigorated, and when the excitement "rubbed away" they could take a new course, restarting the cycle of their own individual professional growth.

More recently, besides classes taken at universities, teachers had the choice of selecting from a slew of professional development activities offered by their own schools or educational systems, which often reflected current interests or trends within the system (Little, 1989). These activities did not attend exclusively to mathematical content knowledge but engaged teachers in the process of acquiring new techniques for mathematical instruction. Courses could focus, for example, on issues such as implementing new guidelines or standards, collaborative groups, or the use of particular resources to teach specific content topics. These professional development experiences were offered in formats such as workshops, institutes, and seminars, providing teachers with "opportunities to connect with outside sources of knowledge in a focused, direct and intense way" (Loucks-Horsley, Hewson, Love, & Stiles, 1998, p. 87). Although the organization and duration of these initiatives varied, participating teachers, again, often added something new to their knowledge or pedagogical stock, which they could take back to their classrooms.

University classes and courses offered by the school system can both be placed within what Little (1993) calls a "training model" for professional development, characterized as "a model focused primarily on expanding an individual repertoire of well-defined and skillful classroom practice" (p. 129). The focus of activities within this model is placed on the individual and the acquisition of new knowledge. Despite differences in duration, format, and content, all these professional development initiatives aim at teacher gains from engagement with experts or knowledgeable others who set up activities to train teachers in certain skills or techniques. In many of these experiences, teachers are "thought to need updating" (Cohen & Ball, 1999, p. 12), and the courses provide the opportunity needed to fill particular voids.

Professional development opportunities under the training model fit within what Sfard (1998) calls the acquisition metaphor for learning. According to this metaphor, a person who learns something new is acquiring a new concept or procedure, which forms the unit of a knowledge that can be "accumulated, gradually refined, and combined to form ever richer cognitive structures" (p. 5). Knowledge can be explained in terms of "such mental entities as cognitive schemes, tacit models, concept images or misconceptions" (Sfard, 2003, p. 355). According to Sfard, many educational traditions such as behaviorism and constructivism use the acquisition metaphor to conceptualize learning. Therefore, professional development initiatives within the training model, many of which are based on different learning theories, operate under the acquisition metaphor.

Back in 1993, Little noted that the training model for professional development did not fit ambitious views of teaching and schooling. She noted that five aspects of

education present in the late 1980s (content reform, equity and diversity, new assessments, innovative approaches to the social organization of schooling, and the professionalization of teaching) required a new perspective for the professional education of teachers. At that time, Little suggested that professional development should include, above all, more collaboration and networks in the context of teaching.

2.2. *A Practice-Based Model of Professional Development*

Nowadays, in many schools, professional development takes very different formats from those used within the training model. While teachers engaged in professional development activities may continue to take courses offered at nearby universities or within their educational system, they can also be found observing in their colleague's classrooms, meeting after school to talk about the design of a particular lesson, discussing students' responses to a problem, working with a more senior colleague to look over new mathematics materials, or meeting at an educational agency to examine test scores with colleagues from other schools in the area. In most of these activities, teachers are working collaboratively on a variety of activities linked to the context of teaching, as Little (1993) recommends. Furthermore, the activities in which teachers are engaged are closely related to classroom practices.

The idea that professional development needs to be strongly based on mathematics teachers' professional practices is one of the current givens among professional developers (e.g., Cohen & Ball, 1999; Cochran-Smith & Lytle, 1999). In the professional development of mathematics teachers, there is a current call for anchoring professional development discussions on the daily activities of mathematics instruction. Therefore, professional development conversations among teachers are focused on the different aspects of knowledge that is specific to the work of teaching mathematics. It is expected that teachers will have opportunities to participate and reflect on their work as they aim to grow professionally. The text of teaching serves as context for teachers to learn about the specific aspects of their labor and reflection is expected to increase teachers' awareness of their practice, allowing them to make thoughtful decisions in the immediacy of classroom work.

Many of these new approaches to professional development fit a new metaphor for learning: the participation metaphor (Sfard, 1998). Under this metaphor,

> the permanence of *having* gives way to the constant flux of *doing*. While the concept of acquisition implies that there is a clear end point to the process of learning, the new terminology leaves no room for halting signals. Moreover, the ongoing learning activities are never considered separately from the context within which they take place (p. 6).

2.3. *From Training to Practice-Based Professional Development*

In this paper, we claim that many of the changes and current approaches to professional development reflect not only the educational changes noted by Little (1993) but also a significant change in our understanding of what constitutes learning. Borasi & Fonzi (2002) recommend that initiatives for the professional development

of mathematics teachers should "be informed by how people learn best" (p. 30). Therefore, new approaches to learning lead to new organization of professional development initiatives for teachers, and the move from an acquisition to a participation metaphor brings changes to the ways in which the professional development of mathematics teachers is conceived and implemented. In contrast to the training model of professional development, we characterize the practice-based model as focused primarily on expanding teachers' participation in the practices of what constitutes teaching. We contend that a move from the training model to the practice-based model for the professional development of mathematics teachers can be understood as a consequence of the move from the acquisition to the participation metaphor for learning. Furthermore, this move is already happening in the design of many professional development initiatives.

Our discussion begins with a detailed presentation of the learning theory that supports the participation metaphor for learning, introducing ideas such as practice and communities, based mostly on the work done by Jean Lave and Etienne Wenger. We connect this perspective on learning to current initiatives in the professional development of mathematics teachers. We discuss some of the professional development initiatives presented during the Study Conference and show how these endorse a participation metaphor for learning. As we present our ideas, we aim at showing that the participation perspective on learning is relevant in the organization of professional development initiatives, moving mathematics teacher educators from a training model of professional development to a practice-based model. We conclude with a brief discussion of how research in mathematics teachers' professional development can find new ways for raising questions for further investigation.

3. On Learning

Teacher professional development deals in its essence with learning, and learning should be the key issue when analyzing teacher professional development. Providers of professional development should start addressing first or foremost the issue of learning and setup the key ideas that support their approach. We want to emphasize practice and experience in producing learning. Therefore, we discuss the relationships between practice and learning, taking a particular stance on communities of practice, which we believe has started to bring new insights into the field of teacher professional development.

Under a situated perspective of research on learning, emphasis is placed on the relationships between learning and the social practices in which learning occurs. For the purpose of discussing mathematics teachers' professional development we assume that instead of defining learning as the acquisition of knowledge of a propositional nature, learning is conceptualized as being situated in forms of coparticipation in the practices of teachers. Additionally, instead of raising the kinds of questions that put emphasis on specific cognitive processes and asking which types of conceptual structures are involved in learning, the relevant issue turns to be

which types of social practice offer the contexts in which (specific) learning takes place—and thus professional development occurs.

We claim that we need to bring in the idea of community because associating practice and community can be helpful in yielding ways of discussing the concept of practice (putting aside within the scope of this chapter concepts like culture or activity) and in defining a special type of community—the community of practice (Lave & Wenger, 1991). However, it should be stressed that a community of practice is not just a group of people who happen to be interacting in a given setting. For example, teachers from many schools who come together to attend a series of workshops may not constitute a community. Nonetheless, teachers teaching in a school in most cases constitute and act as a community of practice—even if they do not recognize or refer to its mutual engagement or shared repertoire (Wenger, 1998). In order to operate as a community a group certainly needs to have a common general interpretation of what is happening and of what members are intending to do.

One theoretical implication of considering a focus on communities of practice is that one places a very specific focus on people: not people in the abstract, but people acting in specific settings and arenas. Teachers in action participating in school activities and building up their strategies, plans, and lessons are members of a community where knowledge develops. That is why, although we seek to produce knowledge about how to go and create strategies for mathematics teachers' professional development, we base our perspective on forms of educating teachers that rely on the possibility of cultivating communities of practice where teachers' knowledge already exists and is further developed rather than on organizing learning outside the situated practices where knowledge evolves.

What teachers learn with the greatest investment and what frames their forms of professional development is what enables their participation in the communities with which they identify. This goes with the general notion that people do better when the depth of their knowing is steeped in an identity of participation and when they feel they can contribute to shaping the communities that define them both as knower and as professional.

3.1. Practice and Learning

Research on learning shows that we need languages to describe in analytical terms the process of coming to know. That is, for example, what Piaget did and did it extremely well. However, those languages must take into account the complexity of the phenomenon of learning, including dimensions such as:

1. a notion of development as something which occurs within a certain dynamic direction (towards something, possibly not known in full) —a *telos*; and
2. the relationships between teaching/instructing and learning which are at the core of the problems of education both in a general sense and in schooling.

In order to preserve the complexity of discussing learning and professional development it is important to look at learning as a learner. One of those key questions we should raise is what we mean when we talk about practice. Wenger (1998) assumes a concept of practice that is consistent with what we can perceive from other authors in the area of situated learning.

> The concept of practice connotes doing but not just doing in and of itself. It is doing in a historical and social context that gives structure and meaning to what we do. In this sense, practice is always social practice...the concept of practice highlights the social and negotiated character of both the explicit and the tacit in our lives (Wenger, 1998, p. 47).

The notion of practice escapes from the idea of just 'doing something'. What is involved in practice is wider and more dynamic, something that for its social nature intervenes in the definition of social communities of a variety of types. If we interrogate some of our everyday actions we will probably find that they are parts of constellations of practices (Wenger, 1998) that are characteristic of the distinctive social practices that are associated with a variety of communities of practice. Teachers working in the school develop a number of tasks with specific goals and intentions, and this makes participation in the school community rather well defined, with boundaries that teachers do not confuse with what is expected from them when participating in activities with their families. Even if overlapping, the communities are clearly distinct, and what organizes the practices is framed within rather different principles. Not all practices lead to the emergence of communities of practice; there should be sources of coherence that make up a community within which practice seems to be the key element.

Wenger (1998) points to three characteristics of a given practice that he identifies as key interrelated sources of coherence: mutual engagement, joint enterprise, and shared repertoire. However, this does not mean that practitioners are conscious of all of it. In most schools, teachers would probably agree that they contribute to the very same social function of schooling, but it is not surprising that teachers do not immediately recognize forms of mutual engagement or that they do not seem to be able to identify their shared repertoire. We should distinguish between what is expected that teachers see as their own practice and what research could reveal about it. This is a rather relevant topic to support the possibility of the involvement of the teacher as researcher on their own practice.

If we see the practice of mathematics teachers as what makes up their professional community at school, their sense of belonging would be a crucial part of membership. Membership in a community of practice is typically a matter of mutual engagement of participants, but this doesn't mean sameness. People bring diversity to the practice in which they are engaged, and the very fact that they are practitioners in a community helps to build some homogeneity between them, affording opportunities for difference. This balance between diversity and homogeneity helps the community to become established, as participants are thrown into the need to deal with others' competences. The diversity and complementarities of roles of teachers in a community of practice relates also to the partiality of knowledge and of

competence of each participant and is a rather important resource for the coherence of the community of teachers.

The possibility of having teachers working together but lacking a sense of belonging to the community of practice at school is real. Time is a key element to allow for opportunities for negotiation of a joint enterprise, making visible the type of processes that turn possible building and assuming the ownership of the practice. There is a sense of self-appropriation that is characteristic of communities of practice and that relates to the idea of mutual engagement.

> An enterprise both engenders and directs social energy. It spurs action as much as it gives focus.... An enterprise is a resource of coordination, of sense-making, of mutual engagement; it is like a rhythm to music. Rhythm is not random, but it is not just a constraint either. Rather, it is part of the dynamism of music, coordinating the very process by which it comes into being. Extracted from the playing, it becomes fixed, sterile, and meaningless, but in the playing, it makes music interpretable, participative, and sharable. It is a constitutive resource... (Wenger, 1998, p. 82).

In their practice, teachers develop resources (both physical and conceptual) that play a major role in the community, making what Wenger (1998) calls a shared repertoire. As teachers adjust their interpretations of actions and sayings, they negotiate meanings which interrelate and make possible the emergence of a shared understanding of what it means to participate in such a community of teachers. Therefore, the repertoire of the community of practice of teachers in a school can be seen as the set of resources shared by them, and that includes the opportunities to contribute to its definition and development. However, as will be discussed, this repertoire should not be understood as reification of procedures bringing in norms, regulations, and sanctions.

3.2. Learning as Participation in Social Practices

Participation describes the social experience of living in the world in terms of membership in social communities and active involvement in social enterprises. In this sense, participation is both personal and social as it involves the practitioner personally in acting in the world, but it refers to actions that have meaning in the social. One rather important implication of this view is that participation shapes our personal experience as it shapes the practice of our communities, the ability to shape those communities being a relevant aspect of our experience of participation.

In order to address the idea of participation it is important to bring into play the concept of reification. Reification refers to the process of giving form to our experience by producing artifacts (both physical and conceptual) that congeal this experience into 'thingness' (Wenger, 1998)—a process which is central to every practice. As participation and reification are complementary processes in any practice, participation makes up for the inherent limitations of reification (Wenger, 1998). On the other hand, participation is usually essential to repairing the potential misalignments inherent in reification, as reification makes up for inherent limitations of participation. Thus, when too much reliance is placed on reification as

well as in participation, the continuity of meaning is likely to become problematic in practice (Sfard, 1998; Wenger, 1998).

3.3. Learning and Communities of Practice

For Jean Lave and Etienne (1991) the notion of situated learning appears to be a transitory concept, a bridge between a view according to which cognitive processes (and thus learning) are primary and a view according to which social practice is the primary, generative phenomenon, and learning is one of its characteristics. There is a significant contrast between a theory of learning in which practice (in a narrow sense) is subsumed within processes of learning and one in which learning is taken to be an integral aspect of practice (in a historical, generative sense).

According to Wenger (1998), "communities of practice are about content. . .not about form" (p. 229). However, and despite the various forms they can take, Wenger, McDermott, & Snyder (2002) consider that there are three key structural elements of a community of practice: the domain, the community, and the practice.

The *domain* of the practice is what creates a common ground among participants and a sense of the development of a common identity, legitimizing the community through "the affirmation of its purpose and value to members and stakeholders" (p. 27). It is the main source of inspiration for the members to contribute and to participate in order that they make sense of the meanings of their actions and initiatives. However, the domain is not a fixed set of problems or elements: it's an evolving entity which encompasses the evolution of the social world and the community itself.

The *practice* is constituted by a set of "frameworks, ideas, tools, information, styles, language, stories, and documents that the community members share. Whereas the domain denotes the topic the community focuses on, "practice is the specific knowledge the community develops, shares and maintains" (p. 29). Practice evolves as a "collective product" integrated into participants' work, organizing knowledge in a way that becomes useful to participants as it reflects their perspective.

> The concept of practice includes both the explicit and the tacit. It includes what it is said and what it is left unsaid; what is represented and what is assumed. It includes the language, tools, documents, images, symbols, well-defined roles, specified criteria, codified procedures, regulations and contracts that various practices make explicit for a variety of purposes (Wenger, 1998, p. 47).

However, it also includes all the implicit relations between participants, tacit conventions that are held, the intuitions that are part of the repertoire shared by them, the specific perceptions of the practice, and the underlying understandings that frame aspects of the practice. Those are some of the signs of membership in communities of practice and are in fact what makes practice to be sustained, but they probably are not articulated at any stage. Therefore, the concept of practice highlights the social and negotiated character of both the explicit and the tacit in the lives of participants.

Assuming learning as a matter of belonging and participating, the *community* becomes a central element as a group of people who interact, learn together, construct relationships, and develop a sense of mutual engagement and belonging. However, the idea of community escapes the idea of homogeneity, as people interacting create conceptual spaces for participation that tend to bring in differentiation and forms of living the practice that go against sameness.

> [If] long-term interaction creates a common history and communal identity, it also encourages differentiation among members [who] take various roles. . .creating their various specialties and styles (Wenger, 1998, p. 35).

What makes a community of practice unfold and constitute a living thing is its members and the forms of engagement they assume. Although keeping, as part of its definition, many reified and institutional structures and resources, the community of practice cannot be reduced to them. This means that, for example, an institutional boundary does not necessarily create a community of practice.

As we discussed in this section, to consider the participation metaphor of learning within mathematics teachers' professional development, one needs to attend to the coherence of the community and the artefacts the community creates and uses. Furthermore, the relations between the domain of the practice, the knowledge the community shares, and the interactions among participants help establish professional development opportunities that operate within a community of practice. It is within this context that certain professional development activities can be considered as practice-based initiatives.

4. On Learning and Professional Development

4.1. Learning and Professional Development

Professional development has shifted from the individual pursuit of context knowledge and often a solitary reflection on how to connect this knowledge of mathematics with one's pedagogy to an enterprise focused on developing a collective of teachers' mathematics for teaching often focused on a dynamic elaboration of children's mathematical thinking. Nowadays, the performance of professional development is targeted to teachers' classroom practices. Professional development projects are varied in terms of scope, level, number of teachers, and interaction between teachers and the context in which they act. The communities of practice that are established also vary in the nature and interaction between the complementary components of participation and reification. That is, professional development projects in mathematics education involve varied domains, practices, and communities.

There exists a variety of investigations on how teachers learn from practice and continue to develop professionally. As Adler, Ball, Krainer, Lin, & Novotnà (2005) note, most of these studies are small scale ($n < 20$) and use qualitative methods, reflecting the complexity of teacher education research. Research on teachers learning through professional development is inherently complex since

"...it deals not only with the beliefs, knowledge and practices of teachers but also students' beliefs and knowledge, as well as with the interaction between teachers and students, and the interaction between teacher educators and teachers...This [high complexity] increases the tendency to keep the sample small in order to reduce the complexity" (p. 369).

To study the complexity of the phenomena—teachers learning in and from practice and developing professionally—investigations need to involve teachers as active participants with student learning as a central focus. What follows are models of practice-based professional development approaches that have been useful in specific settings for engaging teachers and professional developers in learning. Each of these models can be seen as exemplifying ideas of domains, practice, and community as set out previously.

4.2. Models

The theoretical notions of community of practice are mirrored in the structure of some professional development models implemented in different cultural contexts. Each model involves a community of individuals, sharing cultural specificities at the school and societal levels, who have particular forms of engagement in the professional development sessions and whose topic of focus or domain pertains to specific aspects of their practice (preparation and task, broadly speaking) as teachers of mathematics to learners within formal or informal settings. Participants in these models examine, debate, exchange, modify, expand, and create knowledge about their practice. We present three such models: lesson study, toolkit development, and reflecting on discourse. In each case, when possible, we indicate findings that have begun to emerge within these practice-based professional development initiatives.

4.2.1 Lesson study

One community of practice model that has been influential beyond its original geographical location and cultural context is lesson study. To improve their classroom practice, a community of teachers in Japan uses a lesson study—*jugyou kenkyuu*—format. According to reports by Shimizu (2002) and Yoshida (2002), lesson study is a basic practice in Japanese schools at all levels and in all subject areas and occurs at intra-school and inter-school levels. Lesson studies are the primary means of education for novice teachers and of professional development for experienced teachers. There are two salient outcomes of lesson-study discussions. Participating teachers refine further their teaching skills and also learn a great deal of content knowledge. Stigler & Hiebert (1999) outline an eight-step process of the lesson-study approach and suggest that it provides teachers, the primary driving force for instructional change, with a vehicle to exchange their understanding of problems that students face and to generate possible solutions.

The lesson-study model has influenced professional development in mathematics teaching outside of Japan. Lewis (2002) details a four-phase cycle of actions that comprise a lesson-study model: study, plan, research-lesson, and reflect phase.

Teachers use the findings from these phases to improve the lesson and the instruction, and, if desired, the revised lesson may be taught in other classrooms and further refined. For a community of teachers, according to Lewis, lesson study can be an ongoing method to improve instruction based on careful observation of students and their work. It is instructive to note that a lesson is not conceived as rigid, unresponsive script that teachers are expected to enact without regard to the specificities of teacher or students.

Lesson study has gained considerable currency among mathematics education researchers in the United States as a vehicle for fostering communities of practice among teachers (see, for example, Cavey & Berenson, 2005; Fernandez, 2005; King & Murata, 2005; Stigler & Hiebert, 1999). For instance, based on a one-year implementation in a mid-sized California middle school, King & Murata (2005) report findings from a lesson-study model revised to five steps: goal setting, research and planning, teaching and observing, post-lesson discussion, and revising. They found that the "lesson-study planning meetings were the setting for many rich discussions not only about the research lessons but also their own teaching practice" (p. 17). The community of teachers accomplished their goal of learning how to improve the quality of student mathematical conversations in their classrooms. Ultimately, the process provides a pathway for ongoing improvement of their mathematics instruction.

Stemming from a teacher-development project with teachers of middle school, Hanna (2007) and Powell & Hanna (2006a; 2006b) provide another example of teacher learning and professional development in an adapted lesson-study model. In this model, a professional development community is comprised of university researchers in mathematics education, graduate students, and classroom teachers. The university researchers are referred to as teacher-researchers, the graduate students as graduate-interns, and the classroom teachers as teacher-interns. The professional-development model occurs in three nonlinear, overlapping phases. In Phase 1, teacher-researchers study the development of students' mathematical ideas and reasoning in what is called research sessions. In Phase 2, the model has three modalities for engaging graduate-interns and teacher-interns in reflecting on student thinking: reflection sessions, examination of the mathematical tasks on which participants are invited to work, and description of videotape data of the research sessions according to a particular analytic model (Powell, Francisco, & Maher, 2003). Finally, in Phase 3, one year after the start of the professional-development project, teacher-interns enact their version of the research sessions for which they were observers, engaging a new cohort of student participants.

From their engagement in this professional-development model, Hanna (2007) found four areas in which teacher-interns furthered their practice: pedagogy, mathematics, students' epistemological understanding (Steinbring, 1998), and mathematics for teaching (Ball, Hill, & Bass, 2005). She found that pedagogical conversations dealt with issues such as student behavior, facilitating student discourse, building from student work, and managing time constraints. In terms of understanding students' epistemological status, as teacher-interns conversed about the students' mathematical behavior, they often assessed the mathematical validity of students' ideas.

These conversations usually involved analyzing students' work and evaluating their reasoning. Finally, investigating teachers' mathematics for teaching, Hanna noticed that teacher-interns usually stated their expectations for student learning, identifying the necessary skills for effectively teaching mathematics, explaining their reasoning for pedagogical decisions, and outlining the mathematics of follow-up tasks. In sum, teachers gained knowledge of practice through participation in practice and reflection on practice.

4.2.2 Professional development materials

A model of professional development pertains to packaged programs, or toolkits. These toolkits are often developed in conjunction with one or more communities of teachers who teach at the level for which the materials are targeted. There is variation on the extent to which participating teachers inform and shape the materials, as well as to the targeted pedagogical and social issues. For instance, to address the gap between theory and grounded practice around the issues of curriculum standards, effective instruction, and building teachers' capacity, the Star Schools project developed a set of professional-development materials that it calls a toolkit. In their conference paper, Samantha & Jeff (2005) describe that the toolkit of the Star Schools project embodies the following five features, grounded in the literature:

1. connects professional development around classroom instruction (Cohen & Ball, 1999; Cohen & Hill, 2001);
2. supports examination of local and cultural influences on teaching and learning (NRC, 2001; Stigler & Hiebert, 1999);
3. emphasizes a focus on student thinking and teaching for understanding (Fennema & Romberg, 1999; Wiggins & McTighe, 1998);
4. promotes the deepening of teacher content knowledge (CBMS, 2001; Ma, 1999); and
5. aligns professional development regarding curriculum, instruction, and assessment (NCTM, 2000; Loucks-Horsley et al., 2003).

In providing a broad overview of the toolkit and how with it teachers are engaged in professional development activities, Samantha and Jeff outline one of its sections. In that section, teachers participate in the following.

1. focused discussion on pertinent reading from the mathematics education literature
2. guided dissection of a particular lesson from the TIMMS Video Study, including the following actions:
   a. doing the mathematics task,
   b. anticipating different possible responses of eighth graders,
   c. watching video of the lesson planning and implementation, and
   d. discussing effective pedagogical actions that were evidenced in the video
3. reflection of the section of the toolkit

Interestingly, this project and the toolkit that it developed exemplify cultural and international influences on models of professional development from on country into another. Although the Star Schools project occurred in the Untied States, it incorporated aspects of a specific professional-development model—lesson study (Lewis, 2002)—derived from Japan.

4.2.3 Theory in practice: reflecting on classroom communication and discourse

Depending on cultural and local specificities, professional development models may involve a series of workshops or a course with one or more mathematics educators working with a group or groups of teachers. Working in Denmark, Ejersbo (2005), in her conference paper, reports on her experience working with elementary-school teachers, the aim of which was "to develop both mathematical and mathematical-didactical skills". An element of the work was to encourage teachers to develop a reflective disposition towards imagining the unspoken content of communication between students and teachers, the mental processes that occur in the heads of students and teachers during an interaction. This imaginative reification of unobservable content of classroom communicative interactions is based on Ejersbo's use of Leron & Hazzan's (1997) notion of virtual monologue. The didactical transposition of virtual monologue into a pedagogical tool of reflection on practice led teachers to be aware of the questions they asked students and how they listened to students, and how they attended to classroom communications and noticed new things in interactions in a discourse community.

The notion of a discourse community focuses attention on an aspect of a community of practice that is salient and observable. The notion suggests that the members are in continual dialogue, or "multi-logue", with one another (Hanna, 2007). As stated in the first section of this chapter, the unit of analysis is members of the community discursively participating in a community whose intention is to reflect on classroom practice. This practice includes the mathematics curriculum, student mathematical practice, and teacher pedagogical practice. A discourse community is a group of individuals who through the exchange of ideas, thoughts, and concerns about the learning and teaching of mathematics further their existing knowledge of mathematics for teaching.

The models briefly described previously provide specific examples of community formation and identity and of practices that can promote development in teaching and learning. Discussion of further such models can be found in Chapter 3 of this section.

5. Directions for Further Research

Forces internal and external to mathematics education lead to ever-changing scenarios in classrooms that necessitate new visions of professional development in mathematics teaching. New understandings of how children learn outside of school and in formal school settings and dynamic changes in curriculum and in digital technology

result in needs for instructional retooling. Furthermore, population shifts within and between nations produce different instructional needs of children in mathematics classrooms. These forces combined with a revision of our understanding of what constitutes learning in a professional-development environment require further research on how professional-development activities contribute to teachers' learning.

In their review of literature on pre- and in-service, primary and secondary mathematics teacher education published between 1999 and 2003, including articles appearing in the 1998 issues of the *Journal on Mathematics Teacher Education*, Adler et al. (2005) distinguish five under-researched topic areas:

- *Teachers' learning outside of 'reform' contexts*: Many teachers are struggling to develop their teaching skills in environments where reform is not the dominant issue, but assisting a wide range of learners learn mathematics is. How does the dominant thrust of research on and in reform contexts help to understand this? Some critical issues flow from this.
- *Teachers' learning from experience*: We know much less than we should about *what* teachers learn from experience, *whether* teachers learn from experience, and *what* supports learning from experience. Teachers spend most of their time teaching. We understand far too little about what helps some teachers develop from their own teaching while others do not.
- *Teachers' learning to directly address inequality and diversity in their teaching of mathematics*: We know far too little about teachers' learning to directly address inequality and diversity within their teaching of mathematics, and here we include culture, gender, language, socio-economic status, and mathematical background.
- *Comparisons of different opportunities to learn*: We lack comparisons in the field that compare different opportunities to learn. How does one approach to helping teachers to learn mathematics compare with another? We have studied these sorts of comparisons much less.
- *'Scaling up'*: We know little about what happens when programs spread to multiple sites. We have also studied less what it means to scale up or what it means to extend a program that has worked in one setting to another setting—what works, what goes wrong, what designers need to know and think about (p. 376).

These broad topic areas lead specifically to more questions about teacher practice. As Adler et al. (2005) notes, more research attention is needed on issues pertaining to whether and what teachers learn from experience and how to support such learning. We believe such research will help us better understand practice-based models for professional development as well as the role of communities of practice in changing teachers' knowledge.

In a meta review of literature on professional development for mathematics teaching, Sowder (2007) explores what it means to prepare teachers of mathematics and to provide them afterward with professional learning opportunities so that they can help their students learn mathematics successfully. Her concern is to understand what research suggests for teaching mathematics far beyond procedural knowledge and implications for the design of programs of teacher education and development.

She explores questions ranging from "What are the goals of professional development?" to "How do teachers learn from their professional communities about teaching mathematics?" to "What can be learned from research on teacher change?" Her examination is rather comprehensive, and from it one can identify areas needing further research and formulate useful research questions. Here are six salient areas and questions relevant to issues of teachers learning in and from practice in professional development activities:

- Reflection is necessary for effective instruction. However, reflection rarely drives instructional moves. Instead, teachers must continuously observe students and continually make moment-by-moment inferences and decisions. How can the need for time to reflect be addressed?
- Communities of practice established in schools are said to lead to the possibility of individual transformation as well as the transformation of the social settings in schools. How do the transformations of individual teachers and school community occur? How do these transformations influence teacher practice and student learning?
- Much remains to be understood about how teachers' participation in regional and national organizations affects their practice and student learning.
- There is a need to cross borders between research and teaching, connecting teachers with research and researchers with teaching. What are the effects on teacher practice and student learning when teachers and researchers mutually identify and undertake meaningful research?
- What are effective characteristics of multi-year, coherent programs of professional development?
- How are identities as mathematics teachers affected by teachers knowing mathematics and having the associated pedagogical knowledge to teach mathematics?

In this chapter we have looked at the concept of teachers' professional development from a point of view that focuses on participation rather than the acquisition of knowledge. This approach is rather demanding in methodological terms, as researchers traditionally seek precision, clarity, and simplicity in conceptualizing and operationalizing theoretical frameworks, yielding to a concern for maximum definition. However, as Chapter 1 in this section has emphasized, in studying teachers' professional development one should preserve the complexity of that phenomenon and consider learning, thinking, and knowing as relations among teachers in activity in, with, and emerging from their socially and culturally structured worlds. Approaching the notion of professional development from this stance assumes engaging in practice–theory discussion and analysis using teachers' participation in the lived-in world as a key unit of analysis. Participation of teachers in daily practices is based on situated (re)negotiation of meaning and appropriation of specific repertoires that constitute what we call a mathematics teacher. Thus the focus of discussion goes through the issue of quality and relevance of these repertoires and the strategies to engage teachers in practices that entail them.

The teacher is seen as a practitioner, a newcomer who is transforming into an old timer "whose changing knowledge, skill, and discourse are part of a developing

identity—in short, a member of a community of practice" (Lave & Wenger, 1991, p. 122). This perspective means that knowing is inherent in the transformation of identities and is located in the relations among teachers, teacher educators, and their practices. However, it is also constitutive of the social organization of communities of practice.

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Theme 2.3
Tools and Settings Supporting Mathematics Teachers' Learning in and from Practice

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1. Overview

The purpose of this chapter is to present a synthesis of the papers from Strand II which address the tools, dynamics, tasks, contexts, and learning settings that can be mobilized for pre-service and in-service mathematics teacher education. Within this focus, we have identified four topics around which our chapter is organized. In the first section, we deal with tasks for mathematics teacher education, including mathematical problems and activities, which are offered to teachers as opportunities for them to deepen their knowledge of what they have to teach to students and how they can teach this. These tasks are at the heart of mathematics teacher education and determine what teachers are learning, along with several working forms, dynamics, and contexts. Closely connected to the tasks is the topic that is addressed in the next section, the analysis of instructional episodes. These episodes include narrative cases, video cases, and lesson studies. They all provide opportunities for teachers to study and reflect on teaching-in-action. The last two sections tell us more about the context in which teachers' learning takes place. The former deals with *teachers' learning communities*, addressing teachers as learners in communities that constitute an environment in which the participants share experiences, meanings, knowledge, lessons, and stories about the school's practice. The latter describes e-learning in mathematics teacher education and confirms again the power of communities even when these communities are virtual.

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2. Features of Tasks for Mathematics Teacher Education

Teacher education aims at transforming prospective and practicing teachers from novice perspectives on teaching and learning mathematics to more professional perspectives for dealing with the challenges that teaching mathematics presents. This transformation occurs most advantageously through engagement in tasks that foster knowledge for teaching mathematics. Such tasks play a critical role in the learning offers that can be made to participants in various teacher education contexts and settings. We define these tasks as the problems or activities that are posed to teacher education participants. The tasks might be similar to tasks used in classrooms (e.g., the analysis of a graphing problem) or distinctive to teacher education (e.g., an analysis of a videotaped lesson or curriculum material).

Tasks play a significant role in teaching and learning (Krainer, 1993; Sullivan & Mousley, 2001; Zaslavsky, 2005). Hiebert & Wearne (1993) maintain that "what students learn is largely defined by the tasks they are given" (p. 395). This view may be extended to any learner, including prospective and practising teachers. According to Kilpatrick, Swafford, & Findell (2001), "[t]he quality of instruction depends, for example, on whether teachers select cognitively demanding tasks, plan the lesson by elaborating the mathematics that the students are to learn through those tasks, and allocate sufficient time for the students to engage in and spend time on the tasks" (p. 9). Sierpinska (2004) argues that the design, analysis, and empirical testing of mathematical tasks are among the most important responsibilities of mathematics education.

Stein & Smith (1998) suggest a three-phase framework of mathematical tasks used in classrooms for analyzing mathematics lessons. Their framework provides a tool for describing how tasks unfold during classroom instruction, as well as for highlighting the significant influences tasks have on what students actually learn (Henningsen & Stein, 1997).

There is a consensus that a key issue to be addressed in mathematics teacher education is the learning of mathematics. In several models accounting for learning and teaching (e.g., Jaworski, 1994; Steinbring, 1998; Zaslavsky & Leikin, 2004) engagement in meaningful and challenging mathematically related tasks is an important component. In Jaworski's (1994) teaching triad, as well as in Zaslavsky & Leikin's (2004) extension of the triad for mathematics teacher educators, the important role of the task is expressed in the demand for mathematical challenge. In Steinbing's (1998) model of teaching and learning mathematics as autonomous systems, as well as in Zaslavsky & Leikin's (2004) extension of it, solving problems and reflecting on them are critical elements. Zaslavsky (2005) points to the dual nature of tasks from a mathematics educator standpoint: on the one hand, tasks are tools for facilitating teacher learning; on the other hand, through a reflective process of designing and empirically testing tasks, they turn into means of enhancing learning of the facilitator. Zaslavsky (ibid.) offers a detailed account of how a task for mathematics teacher education, both pre-service and in-service, evolved through an iterative process of reflection and how her own learning as teacher educator evolved through this process.

Zaslavsky, Chapman, & Leikin (2003) make an attempt to articulate what may be considered productive tasks for mathematics teacher education. They consider most worthwhile in-service and pre-service activities as problem-solving situations—combining mathematics and pedagogy—which engage participants in "powerful tasks" fundamental for teacher development (e.g., Sullivan & Mousley, 2001; Krainer, 1993). Accordingly, tasks should offer in engaging and challenging ways mathematical and pedagogical problem-solving situations, in which relevant issues are addressed, including sensitivity to learners and reform-oriented approaches to management of learning (Jaworski, 1994). Zaslavsky et al. (ibid.) "expect powerful tasks to be open-ended, non-routine problems, in the broadest sense, that lend themselves well to collaborative work and social interactions, elicit deep mathematical and pedagogical considerations and connections, and challenge personal conceptions and beliefs about mathematics and about how one comes to understand mathematics" (p. 899). Moreover, worthwhile tasks should present challenges to the facilitator as well as to the learners (perhaps with some modifications and adaptations). By this, both educators and learners may have the opportunities to encounter and reflect on very similar experiences.

Many mathematics educators share the view that teaching is strongly influenced by a teacher's personal experiences as a learner (e.g., Stigler & Hiebert, 1999; Zaslavsky, 1995). Thus, many tasks offered for teacher education make affordances for teachers to experience the kind of mathematics and pedagogy that they are expected to offer their students. In-service teacher educators are usually rather flexible and "have a considerable latitude in terms of defining their curricula" (Cooney, 2001, p. 15) and do not face the same time and other constraints that teachers face in their practice. Consequently, tasks enhancing professional growth vary to a large extent with respect to their nature, content, and focus.

Zaslavsky et al. (2003) offer examples of six types of mathematical-related tasks which they consider "powerful tasks" and provide some indications for their potential contributions for teacher learning. Note that these types of tasks lend themselves particularly well to teacher workshops and settings in which a teacher-educator takes a leading role in designing activities and offering them to teachers. These examples involve the following features of tasks for teacher development:

- Dealing with uncertainty and doubt (e.g., resolving cognitive conflict)
- Rethinking mathematics and considering alternatives [Orit Zaslavsky][1] (e.g., considering alternative definitions)
- Engaging in multiple approaches to problem solving (e.g., open-ended problems)
- Identifying mathematical similarities and differences (e.g., through sorting tasks)
- Developing a critical view of the use of educational technology (e.g., learning to appreciate the merits and limitations of "free search" vs. "structure construction")
- Learning from students' thinking (e.g., by becoming familiar with and aware of students' potential responses and creative ideas)

[1] The references in brackets refer to the boxes that are included in the main text.

Orit Zaslavsky

Task: Considering alternative definitions of a square

(Based on Shir & Zaslavsky, 2001; Zaslavsky & Shir, 2005) This task served as the basis for several workshops with secondary mathematics teachers. Teachers received eight equivalent statements describing a square (note that they were not told that these statements were equivalent). They were first asked to individually determine, for each statement, whether they accepted it as a definition of a square. Then they formed groups and discussed their viewpoints and preferences from both mathematical and pedagogical perspectives.

Following are the statements the teachers were given:

1. A square is a quadrangle in which all sides are equal and all angles are 90°.
2. Of all the rectangles with a fixed perimeter, the square is the rectangle with the maximum area.
3. A square is a rhombus with a right angle.
4. A square is an object that can be constructed as follows: sketch a segment, from both edges erect a perpendicular to the segment, each equal in length to the segment. Sketch the segment connecting the other 2 edges of the perpendiculars. The 4 segments form a quadrangle that is a square.
5. A square is a quadrangle with diagonals that are equal, perpendicular, and bisect each other.
6. A square is a rectangle with perpendicular diagonals.
7. A square is the locus of points for which the sum of their distances from two given perpendicular lines is constant.
8. A square is a regular quadrangle.

This task evoked much debate regarding the nature and role of a mathematical definition that drew on teachers' numerous conceptions of necessary features and purpose of use. Clearly, it was a unique experience for many of them—the mere idea of having a choice of what definition to choose and relating it to context was new to them. In addition, considering the alternative statements led them to employ deep mathematical considerations in the course of examining whether the statements are equivalent. Teachers became aware of mathematical subtleties associated with conventions, arbitrariness and freedom of choice. They also appeared to develop an appreciation of the value of such (open) task, and its potential for classroom implementation (Zaslavsky, 2005).

In spite of the significant role tasks play, so far there have been relatively few publications focusing on (mathematical-related) tasks in mathematics teacher education. For example, an examination of articles published in *JMTE* in the past decade indicates seven articles that explicitly address in detail and offer descriptions and analyses of teachers' ways of dealing with tasks facilitated by teacher educators. Interestingly, of these seven articles, three deal with engagement of teachers

in case-study or practice-based tasks involving analyses, construction, and/or reflections on real or imagined cases (such as described in the next section of this paper) (Lin, 2002; Masingila & Doerr, 2002; Walen & Williams, 2000); two engage teachers in sorting tasks calling for examination of deep structures—in which they were required to offer various ways of sorting of either mathematical objects (Zaslavsky & Leikin, 2004) or mathematical tasks for middle-school students (Arbough & Brown, 2005); and the other two articles use a specific classroom mathematical problem (or set of problems) to raise teacher awareness and enhance their content and pedagogical knowledge by implementing the task and considering alternative ways of unfolding it in the classroom—one used a sequence of modelling tasks (Doerr & English, 2006), while the other used a seemingly real-life geometrical task (Fried & Amit, 2005).

In the Study Conference, held in Brazil in 2005, there were two papers that provide examples of some of the types of mathematical-related tasks for teacher education indicated previously. One of these papers (Gilda de La Rocque Palis, 2005) describes an experience that concurs with the last type of tasks described previously, that is, learning from students' thinking [Gilda de La Rocque Palis]. The second paper (Zaslavsky & Lavie, 2005) can be seen as an attempt to make sense of teacher practice and consider alternatives, that is, alternative instructional examples for a given situation [Zaslavsky & Lavie].

Gilda de La Rocque Palis

Activities based on student work

This paper describes structured activities in which secondary teachers engaged collaboratively, focusing on real students' responses to a mathematical problem. Activity 1 is the following:

1. Individually: Answer the question and hand in your resolution: "Let r be the line $y = 3x - 1$ and P the point (1/2,3/5). Decide if P belongs to r, is below or above it".
2. Individually: Give grades to students whose copies were handed out.
3. Whole group: Construct a table showing how many teachers gave grade X to student Y, for all X and Y. Analysis of the table. Free discussion and justification of given grades.
4. In small group: Carry out a new analysis and grading of the same works.
5. Whole group: Compare criteria and grades given by the small groups.
6. Homework: Bring a written account of what you think you have learned through this activity.

Teachers were very much engaged in the activity, individually and collectively. The trust that was built within the group was essential to let the participants to expose their difficulties, sometimes basic ones. Teachers could give extremely distinct grades to the same work but group work took care of those differences. They said explicitly that they never have time to grade students'

answers carefully; they have a model and check student resolutions against the model. It seems that some teachers do not even read what does not look similar to what they already expect to see.

Two similarly structured tasks are activities 2 and 3. Activity 2 deals with students' responses to "Choose 4 points (x,y), one in each quadrant, whose co-ordinates satisfy the inequality $y < x+1$" and asks for a lesson plan to address spotted difficulties. Activity 3 asks teachers to analyze students' responses to "Consider a circle C with given center and radius. Decide if a given point P belongs to C, is situated inside the region bounded by C or is outside this region" sorting their strategies and frame of work (algebraic, graphical). Then, teachers should compare students' strategies with some textbook approaches to the same topic.

Through the work with these two activities, we confirmed that although misconceptions about the graphical context abound, it seems that both teachers and students were not much aware of their possibilities and limits. This kind of work can contribute to the discussion of what content and pedagogical knowledge mathematics teachers should have constructed and represents an instantiation of how this construction may happen (Palis, 2005).

Orit Zaslavsky & Orna Lavie

From a teacher's use of an instructional example to a task for teachers

As a result of a genuine classroom event concerning a mismatch between a teacher's intention in using a mathematical example in her lesson and a student's response to it, indicating such mismatch, the teacher had to alter her example to accommodate the student's thinking. This authentic classroom event was used by the authors as a trigger for in-service teachers to engage in a discussion of what would be an appropriate example in such a case. The teachers generated several examples, and considered each one along its merits and limitations. Through this process they became aware of and began articulating the complex web of considerations underlying choice of instructional examples (Zaslavsky & Lavie, 2005).

There is now both a growing body of literature and substantial interest in tasks presented to prospective and practicing mathematics teachers by teacher educators that have been found to be effective in addressing specific aspects of mathematics teacher education. This is manifested in a special (triple) issue of *JMTE* (Vol. 10, 4-5-6) dealing with the nature and role of tasks for mathematics teacher education. This issue as well as an AERA SIG/RME symposium on this theme (held at the 2007 meeting in Chicago) indicate a shift towards a recognition of this field as a noteworthy part of teacher education. In the next section we will discuss two particularly powerful kinds of professional learning tasks—video cases and lesson studies.

3. Analysis of Instructional Episodes

Teacher educators, professional developers, and researchers have recently taken great interest in the development and facilitation of practice-based approaches to mathematics teacher education (Smith, 2001). In this approach, teachers engage in activities that are deliberately situated in "the work of teaching"—activities that resemble or replicate components of teachers' daily work, such as planning for mathematics instruction, analyzing student work, and viewing and discussing instructional episodes. At the heart of practice-based approaches to mathematics teacher education is an attempt to assist teachers to develop the knowledge they need in their classroom practice by engaging them in tasks and situations that embody the complex interactions that occur in their classrooms. Towards this end teachers engage with tasks that embody authentic aspects of instructional practice and that allow teachers to access, utilize, and develop knowledge of mathematics content, pedagogy, and student learning simultaneously (Ball & Cohen, 1999). Advocates for this approach argue that learning experiences that are highly connected to and contextualized in professional practice can better enable mathematics teachers to make the kinds of complex, nuanced judgments required in teaching (Ball & Bass, 2003; Gal, 2005; Gal & Linchevski, 2005; Little & McLaughlin, 1993).

Using the work of teaching as a central resource, practice-based approaches attempt to coordinate and link different facets of teacher knowledge to each other and to the settings in which the knowledge is used. In traditional approaches to teacher education, the knowledge domains of mathematics content, mathematics pedagogy, and student thinking tend to be treated separately. In particular, teachers often take some specific courses to learn mathematics, different ones to learn pedagogy, and others to gain information about how students learn. Moreover, the content of the mathematics courses is often provided apart from any deep consideration of its use in the work of teaching. One consequence of such a treatment of knowledge is that the learner is burdened with the responsibility for making the needed connections across domains and recognizing the settings in which the knowledge could be appropriately used. In contrast, in practice-based approaches, the knowledge domains are treated as intertwined, and they are tied to settings in which they appear in the work of teaching.

Mathematics teacher educators have developed and utilized several different kinds of stimuli to support practice-based professional development. Of interest here are those that entail the analysis of instructional episodes. Among the most popular approaches of this kind are those involving narrative or video cases of mathematics instruction and those involving the planning and enactment of specific lessons (often dubbed "lesson study"). These resources for teacher learning make the actual work of teaching available for investigation and inquiry (Smith, 2001).

There is considerable variation in how teacher educators have structured and conveyed instructional episodes in their work, but narrative and video cases generally present an entire lesson (or some significant portion of a lesson), providing an edited account of the actions and interactions of a teacher with students as they work on the mathematics at stake in the lesson. In lesson study, teachers typically engage in

collaborative lesson planning, followed by observation and analysis of the enacted lesson.

Video and narrative cases can be deliberately constructed to provoke discussion regarding interactions among the teacher, the students, and the mathematical task in the case, and the way in which those interactions affect students' opportunities to learn mathematics. In addition, cases can provide opportunities for teachers to consider, and sometimes learn, mathematics content as they examine the mathematics featured in the case, participate in analytic discussions about mathematics, and consider how mathematics develops in the lesson depicted in the case.

Video cases have been used effectively in different ways. For example, Gal & Linchevski (2000) were specifically interested in video cases which present identifiable situations that arise in the course of teaching when *the teacher* is unsuccessful in helping the *students* overcome difficulties encountered during studying. These double-foci class situations were named "problematic learning situations" (PLS) and were widely used in pre- and in-service teachers' yearly academic programme, which provided tools to enlarge teachers' awareness of and being able to analyze and cope with such PLS [Gal & Linchevski].

Hagar Gal & Liora Linchevski

Changes in teachers' ways of coping with problematic learning situations in geometry instruction

This paper describes the findings of a study (Gal, 2005) that examines changes in teachers' ways of coping with problematic learning situations (PLS, after Gal & Linchevski, 2000) in geometry instruction. PLS refer to situations in which the student faces difficulty and the teacher has difficulty in helping the student. The intervention was planned as a yearly academic course for pre-service and in-service junior high school teachers of mathematics. The course aimed to enhance teachers' ability to identify, analyze and cope with PLS, expand and deepen their awareness and understanding of students' ways of thinking, and to enhance their ability to retrieve and utilize relevant knowledge while making instructional decisions.

The course combined theoretical pedagogical knowledge with its practical application (using videotaped classroom events involving PLS) to reveal the difficulties of both students and teachers and to illustrate how the theories learned in the course could explain these difficulties. The course was spiral in nature; the participants in the course were first presented with PLS events for which the theoretical material they had already learned could provide an explanation. Then, new theoretical material was presented, after which some of these PLS examples were presented a second time so that they could be re-analyzed in light of the new approaches and so that participants could track their own progress by comparing their earlier interpretations to new ones. At the same time, new PLS examples were presented which could be analyzed in terms of either the old or new theoretical material and over again.

The study was based on three groups: (1) nine B.Ed. students, (2) seven students with a B.Sc. degree in mathematics studying towards a junior and senior high school teaching diploma, (3) twenty-three M.A. students, all in-service teachers. The study analyzed changes in: (1) teachers' awareness of difficulties and ability to identify them, (2) teachers' ability to analyze difficulties by means of cognitive theories, and (3) teachers' ability to suggest effective ways of coping with difficulties.

The findings show a very significant change in the teachers' ability to identify and analyze difficulties, both in laboratory settings and during classroom instruction. Treatment of PLS became an inherent part of their instruction. They were trying to follow their students' thinking processes in real time. Teachers' ability to cope with difficulties in laboratory settings underwent a marked change. The teachers also exhibited an ability to cope with difficulties during classroom instruction, and could analyze and suggest ways to cope with them retrospectively.

The main conclusions were that: (1) the intervention was long enough for most participants to be able to identify, analyze, and suggest appropriate solutions to PLS in laboratory settings; (2) one academic year was not enough in terms of classroom instruction; and (3) overall, there were no essential differences between changes in experienced teachers and novices, or between teachers with a strong or weaker background in mathematics. There were between-participant differences in the extent of change (Gal & Linchevski, 2005).

Another approach is evident in the work of Bao & Huang (2007), who describe how teachers can learn from using multimedia technology to create hypermedia video cases, which record, evaluate, and integrate the crucial elements of exemplary lessons. According to Huang & Bao (2006a) the process of developing an exemplary lesson in the form of a video case study has several advantages: emphasizing professional learning and upgrading teaching, learning ideas, and theories; supporting reflection on or being in action; encouraging revision of design and enactment of new action; and engaging teachers with the process of choosing episodes and creating cases or narratives. Video case studies can include the analysis of lesson episodes from different perspectives and by different agents (acting teacher, colleges, master teacher, students), and this multiplicity of views enhances teachers' awareness and ability to reflect on and improve their own practice (Huang & Bao, 2006b).

A key element in practice-based professional development anchored by video or narrative records of classroom teaching episodes is a well-designed, well-facilitated professional learning task (Ball & Cohen, 1999). Professional learning tasks are complex tasks that create opportunities for teachers to ponder pedagogical problems and their potential solutions through processes of reflection, knowledge sharing, and knowledge building. See the previous section of this chapter for more on tasks used in teacher professional development; tasks associated with narrative and video cases

can be similar to those discussed there but with a specific focus on the particular features of the lesson illustrated in the case.

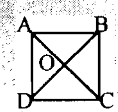
In lesson study, the professional learning task focuses on the design of the lesson and the development of an analysis scheme to determine the extent to which the lesson meets expectations. Gal (2005) studied a spiral method, consisting of teachers' observations of a short video case, followed by an iterative process of reflection on them, study of a theory that can help in analyzing the difficulty presented in the instructional episode, more reflection, study of a new theory, and so on [Gal].

Hagar Gal

Problematic learning situation: Perpendicular lines

(Based on Gal & Vinner, 1997; Gal & Linchevski, 2005).

The following dialogue took place between a (pre-service) teacher (T) and a female student (S) during a geometry class for 9th graders (weak group), where students were asked to examine the properties of a square. The assignment was to check if the diagonals of a square were perpendicular to one another. In the Hebrew mathematical jargon this is expressed by the phrase: the diagonals "cut each other" at a right angle. "Cut" in Hebrew means: divide into two parts, intersect, split! The students were shown the following drawing of a square with its diagonals.



S: (pointing at AC) They "cut" each other here, right? So here it's 90 (points at <ADC).
(She probably means that the diagonal splits the square into two congruent parts. In the triangles obtained as a result, the right angle of the triangle, which is also an angle of the square, is a quite dominant figure).

T: When we speak about perpendicular diagonals...show me the diagonals.
(The teacher tries to locate the source of the difficulty)

S: (Points at AC and BD)

T: That's right. And where do they "cut" each other?

S: (points at diagonal AC and shows that it forms two triangles, ABC and ADC.)
(She probably means that the diagonals split the square in two)

S: (After a brief hesitation): Oh, no, they "cut through" here (points at the four vertices)
(Here, she probably thinks that the question is about the intersection points of the diagonals with the square)

T: Where do these two diagonals "cut each other"?

S: Here (points to O)

T: Well done. Now, when I ask if they are perpendicular to one another, what I mean is that as they "cut each other", are 90° angles formed there (points to O)?

S: Yes! (points to the right angle in each of the two triangles ABC and ADC, i.e., angles <ACB and <ADC.)
(After a while):
Oh, no, these (points to the two other triangles and their angles, i.e., <BAD and <BCD)

T: When I speak about diagonals which are perpendicular to one another, their point of intersection has 90°. Where is that point?

S: (points to O)

T: Now tell me if an angle of 90° is formed between this diagonal (points to AC) and this diagonal (points to BD).

S: (points to <ABC!).

Assignment for the teacher

a. What is your reaction to this section? What are the things that caught your attention?
b. What difficulties for the student arise during the dialogue? Can you explain them?
c. What did you like while observing the teacher?
d. According to the dialogue, what are the difficulties for the teacher?
e. Are you familiar with such a situation? Has something similar ever happened to you?
f. How would you lead the situation if you were this student's teacher? What would you do to overcome the difficulty? (Gal, 2005)

Although there are several similarities between the use of cases and the use of lesson study, there is at least one important difference. In lesson study, the focus is typically on the specific lesson itself, and the aim is the perfection (or "polishing") of that lesson. In contrast, narrative and video cases focus on a specific lesson as an exemplar of a larger class of lessons or as an instance of a more general instructional issue, and the aim is identification and analysis of the more general issue. Thus, learning from lesson study is immediately applicable to practice but its applicability is limited, whereas learning from cases is less immediately connected to practice but its generality affords broad potential applicability.

Although cases and lesson study are certainly different kinds of stimuli for teachers' professional learning, they are in many ways complementary. As Silver et al. (2005, 2006) argue, each approach has strengths and limitations, and the strengths of each address the weaknesses of the other. In particular, case analysis and discussion can be used to build teachers' proficiency with all of the following

intellectual practices and professional dispositions that are needed for successful use of lesson study:

- Treat classroom instruction as an object of inquiry in discussions with colleagues
- Adopt analytic stance toward teaching in general
- Learn to make claims based on evidence rather than opinion
- Attend to general instructional goals and issues
- Consider a classroom lesson as a unit of analysis

Similarly, lesson study may complement and enhance the effects of case analysis and discussion by assisting teachers in becoming more proficient in all of the following intellectual practices and professional dispositions that are needed for instructional improvement:

- De-privatize classroom instruction within a professional community so that others can learn from it
- Adopt an analytic stance toward one's own teaching
- Commit to the steady improvement of teaching
- Analyze general instructional issues in relation to one's teaching
- Consider a classroom lesson as a unit of *improvement*

Both approaches to the analysis of lessons have great potential as the basis for professional learning tasks in practice-based professional development endeavors that are focused on enhancing the knowledge of mathematics, pedagogy, and students that teachers use in their classrooms. In the next section we discuss the nature of teachers' learning communities in which activities such as video cases and lesson studies maximize their potential as learning settings.

4. Learning Communities

The role of the mathematics teacher is to create conditions to support mathematics learning in his or her students. However, the professional development of the mathematics teacher may also be seen as a learning process in itself. Teachers learn in practice, for practice, and from practice. They learn as they design their instruction, looking for new ideas, educational materials, and tasks; as they listen to their pupils' answers, questions and comments; and as they reflect on what happened in the classroom and the suitability of their planning and their actions. They learn as they get involved in projects and all sorts of other activities (Ponte & Chapman, 2006).

Seeing the professional development of the mathematics teacher as a learning process helps in mobilizing current views about students' learning and adults' learning to the learning of teachers regarding their professional roles, and this may bring in new ideas about professional development and learning. It is now widely

recognized that learning is inextricably an individual and social phenomenon that stands on the activity of the individual carried out in a given context (Lerman, 2001). Teachers learn from themselves, from their activity and their reflection on their activity, but such learning takes place in a particular (school) context and in a social environment, interacting with others, notably, students, colleagues, administrators, parents, and other members of the community.

Teachers learn professional knowledge (including knowledge about teaching in general, didactics of mathematics, and knowledge about mathematics as a school subject), learn professional values, learn about their professional roles, and so forth in close connection with other teachers. They begin their learning during pre-service teacher education, when they interact with teacher educators and also with teachers in schools during field work. They continue their professional learning during their professional career, interacting with other teachers in an informal way in their schools, attending professional meetings, reading the professional literature, attending in-service courses and specialized training programs, or getting involved in projects, study groups, or inquiry groups. Learning communities of teachers are special contexts in which teachers learn. These learning communities can be a class of a pre-service, in-service, or specialized teacher education program, or may be a group of teachers from one school who developed habits of working together, or any other group that was constituted especially with the purpose of learning, developing, or inquiring.

The notion of learning community, thus, refers to a group of people with some sort of stability in terms of membership involved in some kind of activity that learn together and, more importantly, learn from each other (Jaworski, 2004). These learning communities may be homogeneous, formed by people with similar professional roles and backgrounds—for example, teachers from the same grade levels from one single school with similar professional experience. However, such learning communities may be heterogeneous—for example, including both primary and secondary teachers, experienced and new teachers, teachers from different schools, or even teachers and other professionals such as teacher educators. Diversity and heterogeneity create more difficulty in finding a common language and adjusting purposes and ways of working but may be of strong value to the work of the group. Different viewpoints, different experiences, and different expertise may make the group more powerful to identify and deal with issues, thus leading to stronger and deeper learning from their members.

There are four key issues in learning communities. One is the purpose of the group and its relation to the individual purpose of its members. The most important condition for one person to learn is that person wanting to learn. Similarly, the most important condition for one group to learn is its willingness to learn. Therefore, we have to pay attention to the purposes with which the groups are constituted, to what brings people to one group and how they identify with the purposes of the group and assume their personal purposes.

This is important since the very beginning but may evolve with time within that group [Fiorentini et al.].

Dario Fiorentini et al.

Learning through collaboration from professionals with different knowledge

The paper addresses an implicit LC, formed by a heterogeneous group of mathematics teachers (the number varied from 7 to 12) and teacher educators (3). The group begun in 1999 and keeps going on. "Each subject takes part in collaborative work with one's own purposes and personal needs and, by means of interchange, also learns more about oneself, about the others and about life in general" (p. 2). The purpose of the group "was to face the challenge of changing school practices towards the construction of the teacher current society demands, which cannot be achieved by school teachers and university educators independently" (p. 1). The interest of the mathematics teachers was in studying and improving their teaching practice and evolve professionally and the interest of the teacher educators was to investigate the teachers' development process in a collaborative context (p. 1).

The dominant topic in this work is the interaction of two different groups of professionals: mathematics school teachers and university math educators and how this interaction is fruitful, originating new knowledge in both parts. This process of generating knowledge involves teachers in "assuming an investigative stance on their practice, developing investigations themselves" (p. 5).

Learning in the group happens through "collaborative work among professionals with different views and knowledge, as well as critical interlocution with studies produced in an academic environment and may contribute not only to addressing such challenges but also to developing teacher autonomy in curriculum management and production of knowledge" (p. 6). This LC shares reflections on: (i) Produced meanings of participants of the group from readings/discussions of papers that discuss new alternatives of teaching in the scholar practice in mathematics; (ii) Narratives/inquiries that had been written by some participants of the group about their own innovative experiences of the practice of classroom. The conditions that made the group have a more fruitful activity, include; "The resignification of participants knowledge took place more effectively when the group studied classroom situations in which student's thoughts when learning mathematics emerged, and they expressed the meaning attributed to mathematics activities" (p. 5). This work shows that shared reflections and meanings, among professionals with different views and backgrounds, contribute to the development of all participants, once they (1) co-produce new significations and knowledge on mathematics teaching and learning and (2) understand their work better, as well as the curriculum, their students and their own role as educators (Fiorentini et al., 2005).

A group that is artificially formed by institutional processes, such as in pre-service mathematics teacher education, may develop as a learning community as its members superimpose on the institutional roles their own learning goals about

practice from practice and by reflecting on practice [Chapman]. A group that starts with weak purposes and relationships may evolve and become stronger, more ambitious, and more productive, and a group with a very good start may decline over time. Any group has its ups and downs—the issue is not how to keep things up permanently but how to renew things when needed.

Olive Chapman

Stories of practice: A tool in pre-service secondary mathematics teacher education

A learning community (LC) is implicit in the activity of a mathematics education course. This LC consisted of the instructor and a homogeneous group of 12 preservice secondary mathematics teachers in one year and 14 in another year. The activity involved writing and analysing stories of secondary school mathematics teaching. The goals of the activity for the participants included: (i) to learn about and from practice through the use of stories of practice, and (ii) to learn how to reflect on practice by unpacking self-stories of practice.

The participants learnt from each other's thinking, written stories, and orally shared experiences. The community provided the voice of the other in order to allow the individual preservice teacher to be questioned and to question or validate through consensus his or her thinking. The stories collectively provided alternatives or possibilities for the preservice teachers to unpack. After initially reading relevant theory and analyzing the story, in the LC, the preservice teachers' role involved sharing their thinking then reflecting on, discussing, comparing/contrasting, challenging, and validating/resonating in their thinking and that of others. For example, they shared their thinking through their analyses of the stories, others reacted to this based on their story by sharing a related aspect of their story to support or counter it, or by offering new stories. They also shared their revised stories of practice with each other and received and provided instructional suggestions. The learning outcome was better understanding of self and practice, for example, as reflected in their revised stories of practice (Chapman, 2005).

The second issue concerns the knowledge that develops from the activity of the learning community based on its shared practices or shared common actions. What do the participants really learn? How do they learn it? In a learning community the negotiation of meanings is a complex process that takes time. Only by being attentive to and engaging with what the others do, feel, and question can participants share a significant learning. The learning process in learning, and their own role as educators [Fiorentini et al.]. Sharing practices and developing together new practices (or new roles in school), learning in close relationship with others, is very recognizable when teachers have to assume new responsibilities at school [Van den Heuvel-Panhuizen & De Goeij].

Marja van den Heuvel-Panhuizen & Erica de Goeij

Offering primary-school teachers a multi-approach experience-based learning setting to become a mathematics coordinator in their school

This paper concerns an explicit LC, formed by a homogeneous group of trainees for elementary school mathematics coordinator. The project had two phases. First a module was developed for future primary school mathematics coordinators (this is a job that did not exist when we started with this module). The module had two focus points: a math-didactical (in this case: gender differences in mathematical knowledge and strategies; what are these differences; what are the consequences for teaching your students; how can you adapt your teaching to the needs of the students) and a professional (how to function as a mathematics coordinator in your school; how to give support to your colleagues). The module was developed by a group consisting of us (university staff) and future "in-service trainers" of the module (staff from teacher education colleges and teacher advisory centres). The draft module was piloted with three groups of future mathematics coordinators who had to comment to the module and had to bring in their own ideas and experiences. After the pilot the draft module was revised. Among other things this meant that many examples from practice were included in the module. Next, there was a schooling for other "in-service trainers" (staff from teacher education colleges and teacher advisory centres) who wanted to give the module at their own location. In the second phase of the project, the organizers of this activity worked together with three "in-service trainers" who had formed each a new group of teachers to give the module. In this second phase of the project the focus was on the future mathematics coordinator in his or her school context. Moreover the mathematics coordinator met each other and discussed their practice. In this case, the purpose of the participants is to learn the objectives of the course — "The course is aimed at enhancing the teachers' domain-specific didactical expertise and the teachers' coaching skills" (p. 2) (van den Heuvel-Panhuizen, & Goeij, 2005).

The third issue is how the learning happens in the group. There are myriad ways, including conducting lesson studies, carrying out collaborative work with other professionals with different views and knowledge, undertaking critical interlocution with studies produced in an academic environment, writing and discussing narratives about classroom practice, reflecting on classroom episodes presented orally or through video records, or conducting small-scale or extended projects. The macro forms of learning communities may vary widely, but the micro activities involve a lot of reading, studying, discussing, reflecting, negotiating, arguing, adjusting, writing, and sharing.

The fourth issue concerns the roles and relationships of the members of the group. The mutual involvement and commitment of the participants to the progress

of the group is a vital condition in a learning community. The learning community is stifled if some members do not feel confident enough to expose their concerns, do not ask for help, and refrain from participation in the group, or if, on the contrary, other members participate "too much", occupying all space, helping others too much or in an improper way, and so forth. A proper style of leadership is a critical element to the working of any group [Ponte & Serrazina]. Leadership concerns the establishment of the group's purposes, plans and the daily conduction of activities. There are always participants that play a more prominent role in one stage or another of any group, but the group itself may establish a collective leadership, assuming the most important decisions after a thorough discussion of the issues, and a distributed leadership for practical activities, assigning specific group members the conduction of a particular activity.

João Pedro da Ponte & Lurdes Serrazina

Understanding and transforming practice: A Portuguese experience

This paper describes an implicit LC formed by ten mathematics teachers and teacher educators, whose work lasted for two years. The group begun as a study group, studying a topic of common interest and later transformed itself in a working group, centred in the production of a book. The members of the group learn from each other as they "write papers and collaborate in discussing their colleagues' papers. The successive drafts were to be sent by e-mail to everyone to be discussed in the following meeting, a process that was used up to the final stage of production of each paper. In this way, the study group transformed itself into a working group (p. 2).

"When the activity was completed, the group carried out a collective reflection addressing what participants thought they had learned, their difficulties, and the aspects that they regarded as most important in the work of the group. The participants indicated that they developed their knowledge and competences, and felt they were growing professionally. They indicate that this activity contributed in a significant way towards knowing better what is involved in the activity of a teacher who researches his or her own professional practice. They also feel that they developed their competency in doing collaborative work and in their communication ability (especially in writing), as well as their self-confidence. Some of them expressed a feeling of professional growth and reinforcement of their reflexive attitude" (p. 3).

Several conditions were critical elements for the success of this LC: "For the participants, the activity was successful because of the collaborative environment, the personal relationships, the group dynamic and the methodology. They also indicate that such collaborative environment and dynamic developed from the style of leadership, largely shared by the group, and the emerging nature of its objectives and working processes" (p. 3) (da Ponte & Serrazina, 2005).

Some natural questions to ask concerning teachers' learning communities in general are the sort of activities related to the work of teachers that help their natural establishment. Two rather important elements of the teachers' work are planning and reflecting on teaching, and this gives rise to interest in models such as lesson study and collective reflection such as the use of video cases, addressed in the previous section of this chapter.

An important variety of a learning community occurs when that community establishes itself as a community of inquiry, that is, when inquiring on some issue becomes part of the purpose of the whole group and of each individual member. Inquiring is a very powerful form of constructing knowledge and therefore of learning [Fiorentini et al.] [Van den Heuvel-Panhuizen & De Goeij] [Ponte & Serrazina]. In fact, it is more and more common to see professionals researching on their own practice, often in collaboration with other professionals and social actors (Jaworski, 2001, 2004; Llinares & Krainer, 2006; Ponte, 2002).

5. Teacher's Learning in Virtual Communities

As has been discussed in this chapter, learning communities cannot be forced, as one cannot impose on the other the need or the desire to learn. In particular, if we talk about professionals, be they pre-service teachers or in-service teachers, it is unlikely that an artificial imposition from the outside will last long. Willingness to learn something is also socially bound, as we are always interacting with others and are always embedded in a culture. A didactical example could be that kids who are born in the United States are more likely to play baseball than kids in Brazil, while the latter are more likely to play soccer, the explanation for this being cultural rather than biological. Likewise, teachers may be drawn to different learning communities depending on the environment at the school where they teach and its surroundings, which may include support from local universities and from the school district.

However, Internet access in settings where teachers find themselves—schools, homes, universities—opens up new possibilities for the formation of communities. With the advent of the Internet, one's interests (in a sport or a topic, for example) need no longer be restricted to one's birthplace, physical location, or culture. As Borba & Penteado (2001) and Borba & Villarreal (2005) argue communities can also be built around common interests as opposed to communities formed for geographical reasons. The transformation of the notions of time and space brought by the Internet have affected teacher education as well and is portrayed in the 15th ICMI Study, albeit somewhat timidly. At this conference, Bairral & Giménez (2005) and Bairral & Zanette (2005) pointed out how teachers from different schools who are interested in geometry use the Internet as a means of building communities to support each other in the teaching and learning of this mathematics topic. Teachers who felt isolated in schools where no one else shared this interest found a "community" in the virtual world [Bairral & Giménez].

Marcelo Bairral & Joaquin Giménez

Dialogic use of teleinteractions for distance geometry teacher training (12–16 years old) as an equity framework

This paper presents a virtual geometric environment for an in-service dialogic course. Such an environment was structured around 6 hypertextual axes/scenarios:

(a) activities which introduce the use of materials forcing the teachers to review their own knowledge on geometry and professional activity,
(b) observations of the role that everyday life plays in the different geometric activities,
(c) reconstruction of cognitive processes of students in class,
(d) observation of the role of manipulative aids for each subject,
(e) organization of summaries of contents, and
(f) continuous self-regulation.

The geometric content was developed in didactical units. A group of mathematics teachers worked on a 50-hour Internet course over six months, using a range of online interactive tools and materials: e-mail conversations, geometric "authentic" tasks, self-regulation inquiries, discussion forum and distributed chats. Three different experiments were done. Semi-structured interviews, text writings and videotaped experiences of teachers' classrooms were used to recognize changes-in-action in geometry by means of their a-synchronous productions (Bairral, 2002). The observations and results are organized in three parts: about the enactive role of interactions, the formative moments, and the new role for trainers (Bairral & Giménez, 2005).

Another paper, Dawson (2005), reports on the challenges of starting mathematics teacher education programs in the Pacific Islands. During his presentation at the conference and in papers that followed this meeting, this author shows how the Internet helped to bridge the gaps between the few face-to-face meetings that could be arranged between different groups of teachers, since travel time between many of the islands is counted in days and not in hours.

The last paper presented at the 15th ICMI Study regarding online education reports on research developed since 1999, focusing on extension courses offered to teachers that involve extensive use of chat for synchronous sessions and e-mail for asynchronous interaction. In this paper, Borba (2005) stresses the social impact that the Internet can have, like in the Pacific Islands, making it possible to connect remote areas of Brazil to mathematics education centers such as UNESP-Rio Claro, in Sao Paulo. The possibility of having a social impact in poor areas, and of taking people out of isolation, seems to be a theme that emerges even in papers outside the ICMI study [Borba].

Marcelo Borba

Internet-based continuing education programs

This paper presents some preliminary research findings regarding the nature of interactions and learning that take place during distance courses in mathematics offered to teachers. Predicated on the belief that knowledge is generated by collectives of humans-with-media, and that different technologies modify the nature of the knowledge generated, the authors sought to understand how the Internet modifies interactions and knowledge production in the context of distance courses. The research was conducted over a period of several years, during distance courses proffered annually from the mathematics department at UNESP, Rio Claro, SP, to teachers throughout Brazil, conducted mainly via weekly chat sessions.

Specific difficulties emerge with the learning of mathematics in such environments. For instance, prior to a scheduled chat meeting with all twenty teachers participating on one of the courses, a problem was posed to them regarding Euclidean geometry. Different solutions and questions were raised by all the participants, but one of the teacher's reflections called our attention. During the discussion, Eliane Cristovão, said: "I confess that, for the first time, I felt the need for a face-to-face meeting right away. . .it lacks eye-to-eye contact." She then followed up, explaining that discussing geometry made her want to see people and to share a common blackboard. While some preliminary findings were presented, more questions were raised: How to provide continuing support to teachers following the course? What mathematics should be taught, once the Internet becomes part of the humans-with-media collective? What are the implications for preparing teachers to teach via the Internet? Others were invited to help seek answers to these questions, as distance education offers new possibilities for teachers to interact with each other and with university professors and researchers over great distances, thus helping to address the disproportionate concentration of knowledge production in certain regions of Brazil (Borba, 2005).

However, the main point that Borba makes is that technology is not neutral at a cognitive level. Similar to the way geometry software transformed the nature of the mathematics generated in classrooms, as shown by research in the 1990s and in this century, the Internet has changed the way teachers interact and the way that mathematics is communicated. Orality and the blackboard shape mathematics in certain ways, and chats and asynchronous interaction through e-mail do so in different ways. In the study paper, Borba (2005) presents the voice of a teacher who complains about the difficulty of doing geometry online due to the lack of a common figure to share with other participants in the course. In a paper presented at Psychology of Mathematics Education 29, which followed the ICMI study paper, Borba (2005b) expands the discussion regarding the influence of a given Internet

interface in the production of knowledge, pointing out that writing, and the kind of writing done in a chat, led in-service teachers participating in a course to develop a kind of linearity associated with their mother tongue, in this case, Portuguese. Santos (2006) takes this argument even further to show how demonstrations about space geometry developed by teachers in chats are developed in parts, in a dynamic way that is strongly shaped by the medium, reinforcing the notion that knowing is developed by humans and by the media that surround them in a given historical moment.

As could be noticed during the 15th ICMI Study, studies of the role of online interactions in teacher education were just beginning. This area of inquiry seems to be flourishing. For instance, Ponte et al. (2007) have studied how different pre-service teachers have different experiences with online supervision of their teacher training. Pre-service teachers felt that online interaction could intensify collaboration but at the same time complained about technical difficulties when using the Internet; they also did not feel comfortable with the way everything is recorded in the "virtual world" and with the time spent writing. Similarly, Ponte & Santos (2005) found that some in-service teachers feel comfortable dealing with mathematics and professional learning tasks in online environments while others do not.

Such a difference could be understood based on studies like those that examine how teachers' identities may change in an Internet environment (Rosa, in progress). This author, employing the notion of humans-with-media, proposes that identity is also shaped by different media that are part of a given collective producing knowledge. In a recently published book, Borba, Malheiros, & Zulatto (2007) raise the conjecture that learning styles are also shaped by different media (Internet, orality in the classroom) and have detailed different needs that different teachers have as they teach and learn in different environments. These authors also illustrate how different abilities are necessary depending on the different Internet interface that is used. Video conferences or chat rooms used as the main interface for a course change the nature of interactions among participants and require different abilities, according to Borba and colleagues.

Other authors do not emphasize the role of the environment as much. For example, in a recent study, McGraw et al. (2007) analyze how non-homogenous groups—composed of in-service mathematics teachers, pre-service teachers, mathematics teacher educators, and mathematicians—analyze multimedia cases. These authors built into their research design possibilities for online forum interactions as well as face-to-face interactions. Participants were able to interact in real time only in their face-to-face meetings and not in the virtual world. In the analysis presented in the paper—in which they focused on the role of different participants as they were exposed to a multimedia case—the role of medium is not treated as relevant.

It is too early to draw conclusions regarding the role of the Internet in teacher education. The 15th ICMI Study showed just a few reports, and work in this area seems to be in its beginning stages even for research groups that are focusing on this topic. It can be said that, at this point, some researchers appear to view the Internet as "transparent", or as not playing any specific role in cognition. For instance, some of the authors view writing in a chat as similar to writing with paper and pencil. Others

believe that the medium is so important that different "units" of collective knowers are formed depending on the medium that is used. It is very possible that there are intermediary positions, but it will be necessary to wait longer to see whether the above distinction is one that will divide the studies on virtual communities, e-learning, and other terms that are being coined. The 15th ICMI Study and the studies that have been carried out on the theme since the beginning of this century show, however, that the Internet can no longer be ignored because, at least, of its social impact.

6. Conclusion

This chapter discusses the role and nature of tasks, contexts, and learning settings in mathematics teacher education. It indicates what may be considered as powerful tasks for teacher development and suggests what may be their desirable features. Those tasks offer mathematical and pedagogical problem-solving situations that address relevant issues in engaging and challenging ways. Particularly noteworthy kinds of professional learning tasks are discussions of video cases that consider the lesson as a unit of analysis and tend to attend to general instructional issues. Another important kind of professional learning task is conducting lesson studies that see the classroom as a unit of improvement and analyse instructional issues in relation to one's teaching in a professional learning context. The measure to which such professional learning tasks promote teachers' learning depends on the activity that teachers generate from them. Such activity depends on a number of factors, some internal to the teacher (his/her interests, concerns, previous knowledge, willingness to get involved), some depending on the wider setting (curriculum and instruction frameworks, nature of contract and school conditions, time available), and some depending on the professional development setting created by the teacher educator (including tasks, resources, schedules, size and composition of the groups, forms of work, and of interaction). A key element of this setting is the learning community—face to face, based in virtual interactions, or a combination of both—that supports each teacher in opening new perspectives about mathematics teaching and learning, challenges their beliefs and conceptions, provides security in attempting new approaches and activities, and promotes effective and reflective change in teacher practice.

It is taking into account this sensitive combination of factors that teacher educators have to carry out their job, identifying what may be important learning goals, assessing participants' readiness, designing tasks, and negotiating working procedures and social relationships to maximize learning opportunities. Designing appropriate professional learning settings for a group of teachers and conducting a professional development activity requires the ability of collecting all the necessary information about the teachers and their contexts, as well as a sensitivity in dealing with often-contradictory emergent phenomena during the activities. Such sensitivity is part of the professional preparation of teacher educators and requires years of reflecting and researching their own practice to develop. However, this sensitivity

may be clearly supported by a general understanding of the relationships among the purposes of the educational activities, tasks, dynamics, and contexts and the roles that different participants (teachers and teacher educators) may assume in these educational contexts.

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Theme 2.4
The Balance of Teacher Knowledge: Mathematics and Pedagogy

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1. Introduction

Teacher education and the professional development of practicing teachers need to provide a sound basis of knowledge for teaching, theoretically but also with strong ties to issues of practice. Although this seems like a common-sense statement, it is harder to make a reality than expected. At least three factors could account for this difficulty: the sheer complexity of the knowledge required for teaching, the interconnectedness of knowledge, and the fact that teachers' knowledge comes from different and in certain cases even contradictory sources. Consequently, (a) there is still a lack of comprehensive and categorical descriptions that frame teachers' knowledge, particularly for content-oriented viewpoints, and (b) there is apparently no broad consensus about the status of that knowledge—is it private knowledge, based on personal experience and only in the personal realm of thinking and acting, or is it knowledge coming from and staying in practice, or is it discursively generated, shared, and general knowledge?

In this chapter we describe aspects of the research on the relationship between teachers' content knowledge and pedagogical practices from various perspectives to address the question, "Is there evidence for a systematic interdependent relationship of content and pedagogy?" Such evidence comes from different sources. One can measure both knowledge facets by the means of questionnaires, by directly observing the teaching practice, and by case studies of selected teachers. Learning about both knowledge facets can occur within the teaching practice as such but also from prospective and practicing teacher education. Moreover, teachers learn from practice—from within the teacher's own practice and from the practice of others as well as from student oral discourse and written productions in their classes.

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2. Domains of Teacher Knowledge

Content knowledge in mathematics is understood to be knowledge of concepts and a fluency of the procedures; however, content knowledge in Shulman's (1986) description goes far beyond knowledge of relevant facts in the domain.

> It requires understanding the structures of the subject matter.... The structures of a subject include both the substantive and the syntactic structures. The substantive structures are the variety of ways in which the basic concepts and principles of the discipline are organized to incorporate its facts. The syntactic structure of a discipline is the set of ways in which truth or falsehood, validity or invalidity, are established (p. 9).

Content knowledge for teachers in mathematics thus contains all the "five strands" contended as the basis of students' mathematical proficiency by Kilpatrick, Swafford, & Findell (2001), which are conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. Also, argumentation and proving is a specific means in mathematics, "to explain why a particular proposition is deemed warranted, why it is worth knowing, and how it relates to other propositions" (Shulman, 1986, p. 9). However, these activities must equally be set into the broader contexts of explaining, communicating, and even modeling (Hanna, 1983; Hanna & Jahnke, 1993) to become an essential and inevitable part of teachers' mathematical content knowledge. In a similar way, basic insights into the history and epistemology of mathematics are necessary ingredients of the content knowledge of mathematics teachers (Fauvel & van Maanen, 2000).

As any description of teacher knowledge must necessarily be broad and multifaceted, beyond teacher content knowledge, two other major domains of knowledge for teaching commonly accepted are *pedagogical content knowledge* (PCK), as described by Shulman (1986), and the more recent revision of PCK as *mathematics knowledge for teaching*, or MKT (Ball & Bass, 2000, 2003). Shulman's conception of pedagogical content knowledge once more underlines the point that teachers' knowledge must be more than just being able to conduct a lesson; he comments that

> within the category of pedagogical content knowledge I include, for the most regularly taught topics in one's subject area, the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations—in other words, the ways of representing and formulating the subject that make it comprehensible to others (p. 9).

PCK therefore is closely connected to content knowledge (in this case mathematics), because the teacher consciously must choose between all the possible representations the subject of teaching provides. It is the focus on representing mathematical knowledge that transforms content knowledge into PCK. However, it is still an open question "whether specialized knowledge for teaching mathematics exists independently from common content knowledge..." (Ball, Hill, & Bass, 2005, p. 45). The MKT is viewed as the mathematical knowledge that is specific to teaching and different from the knowledge needed by other professions, such as research mathematics, engineering, or financial modeling (Ball & Bass, 2000). For example, while the research mathematician strives for mathematical elegance and compression, the

teacher focuses on how to unpack mathematical ideas to make them more accessible to students. Examples of mathematical practices that are specific to teaching include examining alternative solution methods, analyzing their mathematical structure and principles, and judging whether they can be generalized (Ball & Bass, 2003; Ball, Hill, & Bass, 2005; Ferrini-Mundy et al., 2004; Ma, 1999). Recent research suggests that this claim for specialized MKT does exists independently from common content knowledge for primary-grade teachers (Hill & Ball, 2004) and for secondary-level teachers (Brunner et al., 2006; Krauss et al., in press).

4.2 Relationship Between Content Knowledge and Pedagogy

Research is beginning to emerge that extends beyond Shulman's categories to examine the relationship between teachers' mathematical content knowledge and their pedagogical practices. Leikin and colleagues' research findings show that teachers' mathematical knowledge supports better development of their own mathematical and pedagogical knowledge through teaching (Leikin, Levav-Waynberg, Gurevich, & Mednikov, 2006). The results of a German study show that the relationship between content knowledge and PCK are highly correlated (Brunner et al., 2006). In a study of Colombian mathematics teachers' conceptions of their own teaching practices of beginning algebra, Agudelo-Valderrama (2004a) demonstrates the existence of the mutual influence of the teachers' ways of knowing beginning algebra and their conceptions of the crucial determinants of their teaching in their pedagogical practices (see also Agudelo-Valderrama, Clarke, & Bishop, 2007).

A study by Leikin & Dinur (2003) describes and characterizes the factors that affect teacher flexibility in mathematics classes. It confirms that teacher mathematical and pedagogical knowledge and conceptions are the main factors influencing teacher flexibility (consistent with Simon, 1997). In addition, this study shows that other important factors are the teacher's "noticing" and awareness (Mason, 1998; Sherin & van Es, 2005), along with the teacher's beliefs and emotions (Thompson, 1992; Cooney & Shealy, 1997; Sullivan & Mousley, 2001).

Leikin and Dinur differentiate between preliminary and momentary factors that influence teachers' mathematical and pedagogical knowledge. Preliminary factors include primarily the teacher's knowledge and skills—mathematical and PCK (Shulman, 1987), awareness (Mason, 1998), and teachers' beliefs about mathematics and mathematics teaching (Thompson, 1992; Sullivan & Mousley, 2001). For example, a teacher who believes that considering different solutions to a problem is confusing to students will act inflexibly when a solution different from one that the teacher planned is suggested by a student (Leikin et al., 2006; Leikin & Levav-Waynberg, 2007). On the contrary, a teacher aware of the importance of different mathematical solutions to a problem as a means for the development of students' mathematical reasoning may increase a teacher's flexibility during a lesson. Momentary factors are the aspects of the teacher's reasoning and affective reactions in the ongoing moments of teaching, such as the teacher's confusion or curiosity

vis-à-vis the students' unexpected answers and the teacher's ability to understand students' language and notice the potential of their answers (Leikin & Dinur, 2003). The preliminary and the momentary factors are clearly interrelated. For instance, teachers' knowledge may determine and be affected by their ability to notice, the teachers' beliefs may influence affective reactions during the lessons or their momentary decisions about continuing the discussion in a particular direction.

A unique chance to investigate teacher knowledge systematically occurred in Germany due to a national option in the 2003 Programme for International Student Assessment study (PISA) (OECD, 2004). To reduce the explanatory gap between student performance and system variables, a study on the professional knowledge of the teachers of mathematics was initiated (Baumert, Blum, & Neubrand, 2004; Blum et al., 2005; Krauss et al., 2004). Teachers' knowledge was conceptualized in this study, called COACTIV (Cognitive Activation in the Classroom: Learning Opportunities for the Enhancement of Mindful Mathematics Learning), as content knowledge and PCK with specific foci. Content knowledge and PCK were highly correlated (more than 0.60 being a typical correlation coefficient), however, one could empirically separate the two facets, content knowledge and PCK, by the test administered (Krauss et al., in press; Kunter et al., 2007).

Under content knowledge a profound understanding of the topics of school mathematics was perceived (e.g., "Explain why $0.999\ldots = 1$!"). Using Shulman's general ideas, three facets of PCK were contained in this study:

1. A teacher should know about the cognitive potential of mathematical tasks (because mathematical tasks are the most commonly used media to carry mathematical content in the classroom): knowing about students' strategies to solve mathematical tasks, to be able to judge the mathematical and cognitive relevance of the tasks, and having multiple solution paths at hand is crucial for teaching. A sample item is: "Show in as many ways as you can give reasons for: The square of an integer is always 1 bigger than the product of the two adjacent numbers."
2. Knowledge about *students' mathematical cognitions* is necessary for adaptive teaching. Errors and difficulties then can be productive sources for concept building and learning. However, the teacher must be able to recognize these errors and points of difficulty.
3. Knowledge of *mathematics-specific methods of teaching* is necessary, since explanation and simplification are teachers' activities strongly bound to the content itself (Kirsch, 2000). A sample item from COACTIV is: "A student says: 'I don't understand why $(-1)(-1) = 1$'. Please outline as many different ways as possible of explaining this mathematical fact to that student."

The results of a German study show the high correlation between content knowledge and PCK (Brunner et al., 2006). However, it was possible to empirically separate content knowledge from PCK (Krauss et al., in press). The teachers in the academic tracks of the German school system (Gymnasium) seem to have higher degrees of integrated knowledge than the teachers in the non-academic tracks, who showed more separation between content and pedagogical aspects of their knowledge. The influence of teachers' knowledge on students' achievement gains were

found to be completely mediated by the PCK of the teachers, and there was a positive effect; the students' progress in learning was measured by students' results in a German longitudinal component of PISA. The most influential factor of how teachers' knowledge fostered students' achievement was the selection of cognitively activating tasks. Thus, the use of task characteristics proved again to its decisive role as an appropriate means of analysis of teacher knowledge (J.. Neubrand, 2006). See also the discussion of tasks in Chapter 3, Theme 2.3.

In two case studies Agudelo-Valderrama exhibits two Columbian novice teachers conceptions of their practices of beginning algebra (Agudelo-Valderrama, 2004a; Agudelo-Valderrama & Clarke, 2005; Agudelo-Valderrama, Clarke, & Bishop, 2007). She found that the teachers' ways of knowing beginning algebra represented the basis for their pedagogical approaches (i.e., ideas about the why, the what, and the how of their teaching acts). The teachers provided clear-cut evidence not only of their conceptions of beginning algebra, but also of their conceptions of their roles as teachers and the determinants of their practices. Examination of the cases of two novice teachers showed a mutual influence of the teachers' ways of knowing beginning algebra and their conceptions of the crucial determinants of their teaching in their pedagogical practices. One case-study teacher, Alex, who focused on "giving clear explanations of procedures to be followed in the manipulation of given symbolic expressions" (throughout the six months of data collection) attributed "the unsatisfactory results" of his teaching to external factors mainly related to the pupils. In contrast, Pablo, whose concern was "not to tell" and to teach for understanding by promoting pupils' creation of their mathematical ideas, attributed "the lack of success" of some of his pupils to his inadequate knowledge of the teaching of beginning algebra (i.e., internal factors). However, as his knowledge of contextual factors of teaching increased (e.g., knowledge of the expectations of the pupils and their "powerful parents" to cover a list of content items, or the requirements of the school's assessment-report scheme), he started to restructure his teaching in order to align with the institution. His teaching expertise was of less importance compared to his knowledge of how the school functioned. According to Pablo, the parents' requirements were not sensible, but they were powerful people in the school because they paid high school fees, so he believed he had to comply with the parents' expectations. Pablo's knowledge of the structure and functioning of Colombian society and his social dispositions played a strong role in his teaching decisions and his conceptions of his teaching practice. Pablo's knowledge of how Colombian society and the specific school where he taught was more influential on his teaching decisions than his knowledge of beginning algebra and its pedagogy in restructuring his teaching (see also Agudelo-Valderrama, 2004b; Agudelo-Valderrama & Clarke, 2005).

In both cases the teachers' conceptions of the crucial determinants of their teaching greatly influenced their pedagogical decisions. In Alex's case, his conceptions of the role of social/institutional factors acted as justifications that reinforced his conceptions of his teaching of beginning algebra. However, in Pablo's case, his conceptions of social/institutional factors of teaching were more influential in restructuring his teaching than his strong conceptions (knowledge, beliefs, and attitudes) of mathematics. This observation is further evidence of Wilson & Lloyd's (2000)

observation that teachers find it difficult to bring about change despite their strong subject-matter knowledge and commitment.

Agudelo-Valderrama's findings call attention to the critical need to consider teachers' social knowledge as a key component of their practical knowledge, which plays a strong structuring power in their thinking and, therefore, in their pedagogical decisions. She argues that it is necessary to pay increasing attention to the centrality of teachers' social conceptions (i.e., knowledge, beliefs and attitudes) in their thought structures if we are to gain some understanding of the barriers and possibilities of teacher change in pedagogical practices. Agudelo-Valderrama further contends that, in general, the teachers' knowledge and beliefs of the social and educational systems had a strong structuring power in their thinking. Their conceptions (i.e., their knowledge, beliefs, and attitudes) of the social/institutional factors of their teaching represent a key component of their thought structures impinging on their pedagogical decisions, a fact that supports McEwan & Bull's (1991) claim that all knowledge is pedagogical in varying ways. However, "we cannot merely append 'social knowledge' to a growing list of categories" of PCK "because of the fundamentally constitutive nature of social knowledge" and beliefs (Gates, 2001, p. 21).

The findings from this research point to the fundamental importance of sound PCK, which is explicitly bound to the mathematical issues that arise in the class. An ongoing task of teacher education and development programs is to provide knowledge that is properly combined of both components: content knowledge related to the background of the topics taught at school and pedagogical knowledge related to the cognitive aspects of the subject, that is, mathematics. However, Agudelo-Valderrama points out a third component, the crucial role that teachers' social knowledge of institutional and societal expectations plays in the development of their pedagogical knowledge.

3. Learning from Practice

The studies by Leikin, Brunner, and Agudelo-Valderrama all point to how teachers' ways of knowing mathematics affect their pedagogical practices and lead us to consider how practicing teachers learn from their own (Leikin, 2006; DeBlois, 2006) or others' practice (Seago & Goldsmith, 2006; Wood, 2005). See Chapter 2, Theme 2.2, for more discussion about learning from practice.

The main source of teachers' learning through teaching (LTT) is their interactions with students and learning materials (Leikin 2005a, 2006). Leikin (2005b) claims that it is the quality of instructional interactions that exist in the classroom which determine the potential of the lesson to promote both students' and teachers' learning. In this context initiation of interaction by the teacher or by the students, as well as motives for interacting, determine learning processes in the classroom. The motives may be external if they are prescribed by the given educational system, or internal, being mostly psychological, including cognitive conflict, uncertainty,

disagreement, or curiosity. Piagetian disequilibration is the main driving force in intellectual growth or learning. For teachers, unexpected, unforeseen, or unplanned situations are the cause of disequilibration and are the sources for learning. These sources surface via interaction with students and via reflection on this interaction (Leikin & Zakis, 2007). Teachers learn mainly in unpredictable (surprising) situations. As Atkinson & Claxton (2000) contend, many of the teachers' actions when teaching are intuitive and unplanned. Teachers' craft knowledge develops from the transformation of their intuitive reactions into formal knowledge or into beliefs.

Development of new mathematical knowledge takes place at all the stages of teachers' work—planning, conducting, and analyzing a lesson. When planning the lesson, teachers clearly express their "need to know the material well enough" and their "need to predict students' possible difficulties, answers, and questions". At the planning stage the teachers are involved in designing activities that allow them to reach new insights. Hence *new pieces of information* are sometimes collected, and some *familiar ideas are refined* (Leikin, 2005a, 2006). The need to "know better than the students" stimulates teachers' thinking about students' possible difficulties. When predicting during lesson planning, teachers reflect on their own uncertainties and thus resolve their own questions. While conducting a lesson, it is through the interaction with students that teachers become aware of new—for them—solutions to known problems, new properties (theorems) of the mathematical objects, and new questions that may be asked about mathematical objects, and in this way they develop new mathematical connections (Leikin, 2006; Leikin & Zakis, 2007).

DeBlois and colleagues conducted three research studies of six professional development seminars in schools using collaborative research methodology (Desgagné, 1997; Erikson, 1989, 1991). The seminars were held to study pupils' mathematics productions in the context of class history, curriculum, and development of a series of mathematical concepts. Data analysis compared the partners' referents, which revealed the influence of interpretation on the interventions. The first study involved four special-education teachers participating in six seminars over the course of one year (DeBlois & Squalli, 2002). During these seminars, the discussions of special education teachers oscillated between mathematical concepts and the procedural aspects of a mathematical concept (e.g., written number and number) and between judgment of the pupil and evaluation of the pupil's production. When teachers identified a mathematical concept and the possible reasoning of the pupil, their teaching strategies connected to the pupil's reasoning. However, when the reasoning of the pupil, without a conceptual analysis, preoccupied them, a particular teaching strategy appeared; this was trial and error. This kind of analysis did not connect mathematics and pedagogy. This teaching strategy led to a cycle of interventions with no connection to the first pupil's errors. In summary, two aspects emerged as important in this research: creating an understanding of the pupil's reasoning and using error as a component for student learning.

A second study involved three special-education teachers during six seminars (DeBlois, 2003a). For this study, DeBlois proposed an interpretative model of pupils' cognitive activity (DeBlois, 2000) to structure the discussion of pupils' mathematics productions. Teachers were invited to experiment with and adapt the

model. This model, inspired from Piaget's, (1977) reflective abstractive model, identified a variety of components that are coordinated as students work through a solution (DeBlois, 1996, 1997a, 1997b). These components are the *representations* students use to solve problems or complete projects, the *role* they adopt for themselves, the *procedures* they prefer, and the *reflections* that subsequently emerge (DeBlois, 2000, 2003b; Piaget, 1977). The interpretation of pupils' cognitive activities thus works from a number of hypotheses that are developed in relation to these components and to the (reciprocal) coordinations occurring as a result. This model aims to provide some scaffolding to help teachers understand the cognitive activity of the pupils when there is conceptual analysis of the mathematical notion. DeBlois postulates that the ability to understand students' reasoning contributes to the ability to see error as a component of learning and could help teachers gain strategies for working with students who have some difficulties in learning mathematics. This kind of analysis focuses on the interconnection between mathematics and pedagogy.

DeBlois continued to examine the transformation of teaching strategies with a third experiment, in which she studied six seminars with twenty typical teachers in a primary school (DeBlois, 2006; DeBlois & Maheux, 2005). The analysis focused on pedagogy and epistemologies as teachers were investigated using the construct of "sensibility" that emerges from the distinction between situation and environment (Brousseau, 1986; Maturana & Varela, 1994; René de Cotret, 1999). DeBlois observed teachers that were asked to interpret their pupils' errors in mathematics in order to examine the interpretative process and its influence on the choice of teaching strategies in a mathematics class. Sessions were held to study pupils' mathematics productions in the context of class history, curriculum, and development of a series of mathematical concepts. Data analysis compared the partners' referents, which revealed the influence of interpretation on teaching strategy previews. The results provided insight into the preferred teaching strategies with pupils who experience learning difficulties, as well as suggestions on how changes in interpretation transform teaching interventions. When teachers established a link between the task and pupils' procedures, they were sensitive to the familiarity of the pupils' task. Then they considered error as an extension of earlier knowledge and tried to create a gap with original habits. However, when they looked for the gap between pupils' results and results expected, they showed a sensibility to the teaching given and concluded that the pupils had a problem of attention. Thus, they either explained the task again or asked the pupils to read it again. Finally, when teachers identified a gap between what they knew about their pupils and the pupils' production, they were sensitive to the curriculum or to certain elements of the task (type of numbers, type of relation between numbers). In this last case, the error was seen as the product of the interaction between the task and the pupils. At that moment, they considered learning as an interaction between pupils and task. They desired to understand the situation in which their pupils understood the task and they wanted to know the pupils' representations of the task. It is thought that this kind of analysis allowed for an interconnection between mathematics and pedagogy.

DeBlois's research led to a theoretical framework which identifies four components contributing to the interpretation of the logic of pupils (results, pupils'

procedures, progression in pupils' procedures, a gap between what they know about their pupils and the pupils' production), four sensibilities of teachers (teaching, familiarity of the pupil with the task, pupils' understanding, curriculum and characteristics of the task), four kinds of interpretation of the pupils' productions (attention, extension of pupils' procedures, pupils' abilities, product of an interaction between pupils and the task). From this, four kinds of teaching strategies appear: method of working (e.g., present clear directions, grant traces of resolutions, ask to read again the task, circle important words in the task); create a gap with the habits; reconsider certain components of teaching (use a diagram or modeling, exercises, manipulatives); and play with didactic variables (type of numbers).

In a project, Learning and Teaching Linear Functions (LTLF), Seago, Mumme, & Branca (2004) designed video case materials for professional development of mathematics teachers. These materials used other teachers' practice as a basis for examining the mathematics of teaching. The major goals of these materials were to deepen teachers' MKT, specifically related to linear functions. As part of the research and development process in creating these materials, two separate evaluation efforts assessed various aspects of teacher learning. Hill & Collopy (2002) developed and used an external assessment to measure growth in teachers' content knowledge and PCK. They administered pre- and post-seminar assessment using the instrument with a group of twelve LTLF teachers and a comparison sample of ten teachers. LTLF teachers improved in their abilities to algebraically represent problems involving geometric patterns, connect their algebraic representation to the geometric pattern, and compare and link alternative representations of the same linear function. They also were better able to identify potential student misunderstandings that involved using a recursive method for predicting the next term in a sequence. Given the small sample size (N=12), teachers' growth between the pre- and post-tests was not statistically significant. However, equivalent growth did not appear in the comparison group, which provides some assurance that the improvement did not result from retaking the same items over a relatively short time span.

Horizon Research, under the guidance of Weiss and Heck, developed embedded assessments to measure the impact of LTLF materials on teachers' content knowledge and PCK. Embedded assessments in this study meant that the instruments were connected to the actual work teachers do within seminar sessions. In this case, for the embedded assessment instrument administered at the first seminar session teachers were asked to solve a mathematics problem, they reflected on their approach to solving the task and predicted approaches students might use to solve the task. Near the end of the eight-session seminars, teachers were asked to complete a similar process with another mathematics task. The second embedded assessment involved a pre- and post-video analysis task. A comparison group of similar teachers responded to the same tasks. In terms of exhibiting mathematical knowledge pertinent to the teaching of mathematics, LTLF participants were statistically more likely than control-group teachers to increase in their ability/propensity to connect their work on the mathematics tasks back to the pictorial representations in which the task originated (Heck, 2003).

A recent study that investigated teachers' learning from two approaches to mathematics professional development that use classroom artifacts (e.g., student work, transcripts, video clips of classroom practice) to help teachers gain new ways of thinking about their students' algebraic understanding (Seago & Goldsmith, 2006). This work was guided by the formulation of MKT offered by Ball & Bass (2000) and Ma (1999). This perspective focused on the considerable mathematical demands placed on classroom teachers and the kinds of particular mathematical knowledge teachers need to employ in their work. Among the tasks of teaching that require this kind of specialized knowledge include "decompressing" (unpacking) mathematical ideas, analyzing student thinking, choosing representations to effectively convey mathematical ideas, and negotiating mathematically productive discussions. This study focused on changes in teachers' analysis of student thinking, use of representations, and unpacking mathematical ideas.

Goldsmith and Seago (in press) examined the impact of two commercially available professional development programs focused on algebraic thinking: *Fostering Algebraic Thinking Toolkit* (*AT*) (Driscoll et al., 2001) and *Learning and Teaching Linear Functions: VideoCases for Mathematics Professional Development* (*LF*) (Seago, Mumme, & Branca, 2004). These two programs were designed to make use of classroom artifacts to help teachers examine issues related to algebra: *AT* centers on the exploration of algebraic habits of mind, and *LF* focuses on linear functions. Four professional development seminars were conducted, two *AT* groups (both facilitated by Driscoll) and two *LF* seminars (both facilitated by Seago). Seventy-four U.S. middle and high school teachers participated in this study: 49 in the experimental groups and twenty-five as a comparison group. Sixteen case-study teachers (four from each site) were followed more closely.

Results indicate that teachers across both sets of materials learned to be more analytical about student thinking, as evidenced by performance on post-program written assessments. The participants who conducted analyses of video and written student work were more grounded in evidence, more focused on the specific mathematics captured in the artifacts, and more attentive to the mathematical potential of students' ideas (instead of just the correctness of the work) than those of the comparison group (Goldsmith et al., 2006). Analysis of seminar discourse indicates that participants' analysis of mathematical thinking became more sustained, extensive, and nuanced over time, and they developed more differentiated, representation-rich, and flexible approaches to mathematics (Goldsmith et al., 2006; Seago & Goldsmith, 2006) than did the comparison group.

4. Implications for Practicing Mathematics Teachers' Development

The emerging research on the relationship between teachers' content knowledge and pedagogical practices is promising. The studies in this chapter show that teachers can learn PCK in and from practice. Yet more systematic research is needed

to understand the conditions under which teachers learn and how it affects their practice and ultimately their students' learning.

For further research on the relationship of content and PCK for teaching, some issues still remain, even if we already have encountered in this article some indications towards answers. What can one say, theoretically and with empirical evidence, about the structural characteristics of teachers' knowledge? Are there researchable units of analysis which can serve as tools indicating certain specific aspects of teachers' knowledge? What are the ways teachers' knowledge influences their teaching practice? How strong is the evidence about the impact and effects of teachers' knowledge on students' achievement? Finally, there are two questions on the possibilities of teachers' further development or even changing their behavior: What are the essential places where teachers learn; is it teacher education and/or the practice itself? What must occur so that the actual teaching in a class is affected by the knowledge that the teacher has acquired?

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Tenoch Cedillo & Marcela Santillan (National Pedagogical University, Mexico), Algebra as a language in use: A promising alternative as an agent of change in the conceptions and practices of the mathematics teachers.

K. C. Cheung & R. J. Huang (Faculty of Education, University of Macau, China). Contribution of realistic mathematics education and theory of multiple intelligences to mathematics practical and integrated applications: Experiences from Shanghai and Macao in China.

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Lucie DeBlois & Jean-Francois Maheux (Laval University, Canada). When things don't go exactly as planned: Leveraging from student teachers' insights to adapted interventions and professional practice.

Roza Leikin (University of Haifa, Israel). Teachers' learning in teaching: Developing teachers' mathematical knowledge through instructional interactions.

Teresa Smart & Celia Hoyles (The Institute of Education, United Kingdom). A programme of sustainable professional development for mathematics teachers: Design and practice.

Olof Steinthorsdottir & Gundy Gunnarsdottir (Iceland University, Iceland). Analysis of professional development programs in Iceland.

Terry Wood (Purdue University, U.S.). Developing a more complex form of mathematics practice in the early years of teaching.

Learning in and from Practice: Comments and Reflections

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What can I add to such a sum of the quite interesting ideas presented in this strand?

I quite agree with the important points often emphasized in the texts: the importance of taking into account the complexity of the related issues, not only when interpreting research results but also when conceiving a study; the importance (perhaps even the necessity?) of the collective work, whatever it could be, in in-service teachers' development; the importance of the (long) time to make training effective—even if it is implicit in some works; the importance for professional development of taking into account and even mixing actual teachers' practices and knowledge which is not self-sufficient whatever its contents could be. Nothing is missing!

I have particularly appreciated the shrewdness of Chapter 1, maybe even a bit difficult sometimes. The last two questions of Issue 3, under Part C, are particularly interesting, even if no answer can be found in the text; let me add that the glossary is not quite complete (I miss "constructionist", for instance, and do not agree with the complexity definition!). What I retain from Chapter 2 is the (implicit) statement of the necessity of introducing some elements related to the craft in teachers' development, whatever form it takes. Chapter 3 is very clear and the (mathematical) examples are not only very interesting but also necessary—I was waiting for some examples in math to exemplify the general comments! Apart from the classical classifications of knowledge for teachers, the last chapter presents some news trends related to new ways to conceive an access to precise teachers' activities. They are considered as pieces of a specific professional knowledge (for both points of view of researchers and teachers): I particularly appreciate their "full-of-promise" presentation.

Then I go on with my critical effort, putting on my nose my "distrust" glasses...

I wonder first whether the (internal) difficulty for the authors to respect the demand of using the ICMI papers in priority (if not only) does not explain some heterogeneity of the chapters—according to their more or less acceptance of this prerequisite.

In chapter 2 the authors develop all the possible ways of involving teachers in a practice-based development-and particularly on all collective ways, such as community of practices, to realize such an involvement. They insist on the change of training conceptions, moving from the idea of acquisition to the idea of participation for teachers (taken as a metaphor).

R. Even, D.L. Ball (eds.), *The Professional Education and Development of Teachers of Mathematics*, DOI 10.1007/978-0-387-09601-8_19,

It may be explained by noticing that a common feature of all the chapters is the place given to collective work in teachers' development. However, there are some different meanings of this "collective" property, and it is perhaps not sufficient to recognize that some "scenario" may be labelled with this term: it is not sufficient to understand what is "played" in the training nor to ensure further qualities of the training. The difference between lesson studies and case studies, as so well explained in Chapter 3, is a good example.

Let me go on with this important issue. Another point on these "communities of practices" is the following: are we sure that the simple fact of letting teachers work together will permit in overcoming every difficulty? If not and if, as I think, there are issues too complex to be overcome by the mere fact of getting teachers together, what kind of help does someone else have to give and how? It brings me to add some questions on trainers. Some researchers have developed a very interesting point of view, that is, the necessity of a collaboration between teachers and researchers, but is it a necessary and/or sufficient condition for the trainers? The question of the teachers' trainers seems somehow crucial to me, according to the importance given to collective work and to the necessity of good support, which cannot be improvised. This point is perhaps not developed enough in the text.

Some other implicit questions arise immediately when tackling such issues as in-service mathematics teachers' professional development or growth. Let me recall some of them as a means to explain how the different chapters of the second strand deal with them.

First, there are different ways to conceive teachers' growth and training models: one can think of development in terms of changes; another will think in "enriching" terms. Behind this nuance, we find some authors' conceptions of what may be "good" teaching or a "good" teacher, if that exists, and perhaps there is a difference (a gap?) between the two caricatured positions of "you have to do that or that to be a good teacher" or "if you do that as a teacher, then some particular events may occur in the learning of your students". Such variables, tied to trainers' beliefs, may intervene in understanding the development process.

In the same connection, the spreading of materials such as the National Council of Teachers of Mathematics standards may be "taken as shared" by researchers; if not, researchers have to continue studying them thoroughly, for instance, differences between students may lead researchers and then teachers to adapt some of this "advice", and it may have consequences on training programs.

Another difference may emerge from the difference between "change" and "enrich": it is the way that is chosen to start the training and to anchor it to the actual teachers' practices related to practices' theory taken up.

More generally speaking, let us try to adapt Vygotsky's theory of the Zone of Proximal Development (ZPD) to teachers' practice growth. In a simple way, if the training is anchored in teachers' actual practices, we may interpret it as an attempt to draw on these (acquired) practices to reach new ones not too far from the first ones. Pursuing this idea, the intermediate to go on enriching practices may be the organization of some teachers' work on their own precise activities (in Leontiev's sense), with some "metacomment" allowing teachers to enrich their practices. It is

an extension of the use of the classical students' "tasks and activities" to professional development.

Roughly speaking, we can distinguish three kinds of mathematics teachers' activities, which are tied to teacher planning, mainly choices made on content, tasks, and previsions of students' actual working; classroom management, including student enrolment, improvisations, and assessment; and reflections after the sessions, more or less implicitly. Enriching practices may involve engaging teachers in precise activities of each kind and then studying them.

For instance, Delbois' examples in Chapter 4 may be read as a work on teachers' specific activities tied to managements and reflections: it is related to recognition and interpretation of students' mathematical work. The training involves the possibility of thinking (systematically) about this kind of activity and then enhances the ability to relate such interpretation to a more global view of the students' actual failure in the used procedure. It makes the teacher able to intervene more closely with the students' errors according to the individual student's knowledge or lack of knowledge (in a ZPD perspective for students). Delbois lets teachers work as usual and in some way lets them add something to their usual activities, that is, why one can speak of "enriching".

Working on content knowledge, mathematical content knowledge, and pedagogical mathematical knowledge may also give rise to enriched teachers' tasks and activities more related to choice and management, such as "unpacking mathematical ideas" (see Chapter 4). Let me note that it is mainly teachers' work about exercises that researchers trend to "modify", but there are other important teacher activities, such as exposing pieces of knowledge that may be also studied.

There is another implicit question that lends a common feature to all the chapters: the lack of precise (research) proofs when studying training's effects, for instance. Of course it was not the aim of the strand, but maybe some uses of Chapter 1's benchmarks or issues are missing. If recognizing complexity of the issue is another common feature of all these studies, there is a gap between Chapter 1's level of characteristics in the process of developing professional expertise in and from practice and other ones. For instance, the authors could attempt to label some of their works as examples of some of the dyads or questions presented in Chapter 1.

Actually, when addressing the professional teachers' development related to training, this proof question is unavoidable but "impossible to answer". It is related to the fact that teachers' training involves a "three-level" piece of work, with many issues not yet solved for each level: the teachers' training level, including trainers, and training scenarios (models and implementation), the teachers' actual practice level and their relations to the training and the students' learning level, according to teachers' practices. The difficulty of getting proof about the effects of teacher training is related to the more general and well-known difficulty of connecting, for instance, students' learning and teachers' teaching, with all the differences that may occur between classes, students, and so on. However, in our case it is more complicated, as there is a third level. It is then particularly interesting to give some information on how the proofs were obtained.

I have some last questions about the methodology and the (lack of) discussion: my first questions are tied to the collection of the videos: is it so easy to obtain a video with a "problematic learning situation", as in Chapter 3?

More generally, there are questions tied to some precisions, eventually missing.

It is sometimes difficult to know if the word "learning" concerns students' or teachers' learning.

More importantly, I often miss details on the precise unfolding of sessions in learning communities. Who speaks first? Who decides to finish a debate? The fourth issue about kind of questions developed in Chapter 3 is very important in increasing researchers' awareness on the importance of these questions according to the understanding of what may really occur in groups.

At last, in each study there are some limits and also some factors that appear to be variables, related to the number and the range of the studied cases, or to the specificities of the concerned students or teachers, or to the analyzed content, or to the time (length) of the experiment. Although it is always instructive to know them, there are few such nuances or discussions in the chapters. For instance, I wonder whether training needs and constraints are the same for high school and elementary school teachers, who work in different mathematics and pedagogical contexts. Surprisingly, I did not find this variable in the whole strand.

To conclude, if it is possible to add something in the heading, it would be ten lines with the following ideas.

1. The text is a little bit heterogeneous, in accordance with the diversity of the authors' sensibility; it will fit the readers' diversity!
2. According to the present extent of the works, there are some limits and some variables in the research results, tied to the teachers' diversity (see Chapter 1)—further studies have to go on detecting them and completing the range of the research.
3. An extension of the research to mathematics teachers' "tasks and activities" may provide new perspectives, including (and intertwining) different types of knowledge into actual practices (see Chapter 4): the communities of practices become the means to develop such precise professional growth and not an aim in itself (see Chapter 3).

Established Boundaries? A Personal Response to Learning in and from Practice

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I had the privilege of playing an integral role in the the 15th ICMI Study, initially as a member of the study's International Programme Committee and later as a replacement for João Felipe Matos in the planning and running of the second major strand—Learning in and from Practice—at the study conference, which was held in Aguas de Lindóia, Brazil, in May 2005. My responsibility as a strand leader was to facilitate and coordinate one of the four groups in Strand II, which focused on Learning for and in Practice.

For me, the study conference marked a wonderful occasion in which considerable effort was taken to bring together as diverse a group of researchers in mathematics education as possible who were involved in the professional education and development of teachers of mathematics. The programme for the study conference was constructed in such a way as to attempt to maximise discussion and debate around the accepted papers. The choice of a scenic but isolated venue where all participants stayed in the same hotel created opportunities for more intense interaction than usual at both formal and informal levels.

The task of an ICMI study volume is to present a report on the field which serves as the focus of the study. For ICMI 15 the focus was on the professional education and development of teachers of mathematics. It was made clear to participants that the extent to which the study volume draws on the study conference is left to the organisers of the study.

The overview to this group of chapters which represent the work of Strand II (Learning in and from Practice) at the study conference explains that the coordinators of this section have chosen to work with the material that was submitted as papers for the study conference. They have written four chapters that reflect the major themes that came out of the synthesis of the discussion from the four groups. The process through which these chapters attempted to continue the participative model created by the conference is described. All participants from the strand who attended the study conference were invited to become authors, and each chapter was based on an appropriate selection of the papers that were presented at the conference. Care was taken to aim for as much diversity and inclusion as possible.

This was a commendable yet challenging brief, and the coordinator(s) of the section and the section's chapters have done an excellent job in tackling the given task. The themes of the chapters have been appropriately chosen, and while each

R. Even, D.L. Ball (eds.), *The Professional Education and Development of Teachers of Mathematics*, DOI 10.1007/978-0-387-09601-8_20,

chapter has responded to the task of including the various voices of the original presenters in different ways with a varying number of study-conference papers being included in the references, I believe that as a whole they present a most useful and challenging reflection of the field. The chapters also represent a good selection of the themes of the accepted papers.

I have indicated in the title that my contribution to this section comes as a personal rather than a critical response. After reading the four chapters I found that my thoughts kept coming back to a similar theme as the writing successfully transported me back to my memories and notes from the study conference in Brazil. I found myself left with a sense of regret at what I saw as a missing dimension of the conference. This absence speaks to a larger systemic issue and seems to link to the challenges to embracing complexity that are presented at the end of the first chapter. So my insider/outsider dyad got in the way of my given brief for my written contribution, and my insider voice is demanding that I pay attention and try to articulate my sense of loss.

The root of the issue seems to lie in the fact that, as strand leader for Group 2 at the conference, I placed a slightly different emphasis on how I facilitated the process at a crucial stage in comparison to the other three strand leaders, and I can see that it is this slight difference that has caused the problem/opportunity.

Our group leader brief for the strand sessions was to spend the first four sessions in our group using the accepted papers as seeds for discussion without having them presented orally. Three papers were selected for each session, and participants in the group were asked to make sure they had read the three papers before the start of the session. A critical respondent was invited to respond to the three papers and present the group with some questions, after which we split into three subgroups. Each subgroup had an assigned leader whose task was to facilitate discussion and provide a written report of the discussion that took place that had been stimulated by the papers. The final 30 minutes of each of these 100-minute sessions was planned to see a combination report back "with all the people in the working group together,. . .based on the key issues extracted by each of the three groups, with the aim to try to reach some common synthesis for the day". The aim of the fifth working session was for each working group "to draw together the discussion and issues from the previous four sessions. It will be organised at the conference to allow flexibility relating to achieving a good synthesis of the work so far".[1]

For Group 2, I decided to leave as much time as possible for group discussion and so did not bring the sub-groups together at the end of each of the first four sessions. Instead I left the discussion hanging in the hope that the absence of closure for the day's work would allow space for cross-fertilization and further thought about the issues raised. So by the end of the fourth day there was an accumulation of ideas stimulated by the papers and by the ongoing group discussions.

[1] Quoted sections are taken from the document "Organisation of work within Strand 2", which was sent to all Section 2 participants before the conference.

In an attempt to tap into this ongoing discussion, I organised our fifth session using the concept of Open Space Technology (Owen 1990). Through this process, group members were asked to identify the most pressing issue or question that was with them as they came to the end of the three days of paper presentations and discussions. My appeal was that each person push himself or herself and share with us a question/issue that was at the core of his or her work. Each person posted his or her issue/question on the wall and then went through a process of discussion and negotiation in which each participant wrote his or her name on the issue that interested him or her most, after which issues were removed or combined. At the end of this process we were left with a set of five key topics of the moment where there were at least three people who wanted to discuss that particular topic for the rest of the session. The initiator of the topic was given the task of facilitating discussion and also for ensuring that a written summary of the session was submitted to me that same day. Each group then presented their summary to the conference on the final day.

The reports from that fifth session indicate that the topics that were selected as a result of the Open Space process were the following:

- Understanding and supporting "community" and "change" from the inside up.
- Attending to the rational and the practical aspects of mathematics' professional development initiatives is not enough.
- Exploring heart/head aspects of our work as implementers/researchers.
- Respect and trust: building community.
- In-service teachers as learners.

Three of the main memorable points raised in the reports of these groups follow.

- We need to revisit our understanding of community. How do we become involved in the building of a genuine community with those with whom we work? How much attention do we pay to issues, such as the inevitable power differentials that are most certainly present whenever different groups come together? How do we go about developing respect and trust when working with teachers?
- If we really believe that we respect teachers and accept them where they are without imposing our own agendas and by consulting them at all times about what they want, why aren't we succeeding more? Why aren't teachers buying our goods even when these goods seem to be what they have explicitly requested? Surely we have to pay far more attention to the emotional aspects of our work to begin to understand this phenomenon.
- Our work involves us being both implementer and researcher within both public and private spaces. Wherever I am, I tell stories. What counts as a convincing story? What counts as legitimate and valid? If I use these stories as evidence, there is a tension between the evidence I can use as an implementer working with teachers (informal, oral, stories, poetry, metaphor) and the evidence that I provide as a researcher to the public (formal, written, data, evidence). The heart is media for learning, while the head is media for creating knowledge.

These were the Group 2 issues from Strand II that were present in the moment at the end of the study conference that had been stimulated in part by the papers presented but also by the lived experiences of the participants from their involvement in the practice of professional development in mathematics teacher education. I have highlighted in bold some of the terms which seemed to be significant to them in their reports.

As part of my exploration of my disquiet on thinking about writing this response, I decided to use the search facility to explore the number of times these bold words appeared in the study volume's chapters in Section II.

Most of them either do not appear (**agenda, poetry**) or appear only once (**trust**) or twice (**head, heart, respect, emotion**) and then often in a different meaning (for example, **respect** appears twice, in "respectively" and "with respect to"; **power** appears in relation to objects, as in powerful tasks and powerful knowledge; and **stories**, while appearing most commonly of *all* the words, appears in a concentrated space as "stories of practice", which deals with one particular method of working with teachers—not our stories—and as a single option towards the end of a whole range of options, as in frameworks, ideas, tools, information, styles, language, stories, and documents!).

So I notice this absence from the study volume of ideas and discussions that seemed at the time to be vibrant, alive, and at the very core of these Group 2 participants' learning in and from practice! In fact, several of the members of Group 2 did in fact respond to the invitation to volunteer as an author for the study volume and have contributed to these published chapters yet do not seem to have picked up on their conference ideas for their study-volume contributions. What might be going on here?

My hypothesis is that the very different substance of Group 2's focus has something to do with my failure to bring each session to a close with a summary of the day's discussion, and the open-ended nature of the Open Space process gave group members the opportunity to continue working with the questions that were present in the moment rather than return each day to the issues that they had raised in their papers which recorded their thoughts and conclusions captured almost a year before the study conference. The open and inclusive process set up by the study-conference organisers and the choice of venue did the rest and allowed for immediate conversation, which meant that there was time for trust to develop and allow a more informal mode of interaction to come to the fore, where important, less-talked-about issues could be raised.

In thinking about possible reasons for the absence of these words from the Strand II chapters in this study volume, I am reminded by the statement of one of the group participants who was discussing the heart/head topic shown previously. She said that her standing in the field depended on her satisfying the necessary conditions of expertise for publication that resided very firmly in what she called the Researcher/Head dimension (the group later tried to capture this image in the matrix in Table 1). An interesting consequence of this was that she felt that some of her most interesting data and insights could not be brought to public attention, as the manner of collecting the data and the best way of telling its story did not fit neatly into the

Table 1

	Implementer	Researcher
Head	?	"Data" Evidence: quantitative, qualitative
Heart	Stories Metaphors Poetry Analogy	?

Researcher/Head block but had more in common with the Implementer/Heart block. This to me is a serious concern that needs further consideration.

What are the established boundaries (real and imagined) that restrict and confine the voices that have a different story to tell about our research into learning in and from practice? What assumptions and beliefs do each of us have in place as to what the field can and cannot tolerate? How much does the university's priority for "recognised publications" form the types of stories we tell? To what extent do the needs and preferences of funding agencies form our research agendas and the manners in which we report them? Do we believe that something lies outside our field of mathematics education if one can replace the word in mathematics with the word history? What aspects do we lose by taking similar positions?

In this study volume the co-chairs of the study allowed considerable space for exploration and innovation in the way in which they set up the study conference and the possibilities for the study volume. At one stage there was an intention to include material from the implementer side, such as an accompanying CD and writing that took an activity-based focus as its starting point. Neither of these ideas came to fruition. I am sure that they would have been willing to explore some of the ideas that were expressed by the Group 2 summary. Yet none of the participants of Group 2 (including me!) sent in such a proposal.

I have a strong recollection that the empty blocks in the matrix were not empty during the session and have somehow fallen by the wayside. They present an interesting challenge. What informing theories and forms of reporting might fill these currently empty blocks? In particular I have drawn attention to the challenging Researcher/Heart block.

After writing the previous text, I returned to the chapters from Strand II in this study volume and was drawn to the following section from Chapter 1, where the reader's attention is drawn to the fact that, as researchers and teacher educators, we might be facing certain ethical issues that merit consideration. I offer a slightly abridged version of those questions:

- How do we track our own complicity in the phenomena that we study and report on?
- How are our assumptions concealed and/or exposed by our languaging practices?
- What is the role of different vocabularies and forms of storying in helping us to think/act differently with regard to our roles as researchers, teacher educators, teachers, and mathematics knowers?

I have a current favourite poem that for me highlights these dilemmas:

Don't establish the
boundaries
first
the squares, triangles,
boxes
of preconceived
possibility,
and then
pour
life into them, trimming
off left-over edges,
ending potential:
let centers
proliferate
from
self-justifying motions!
(A. R. Ammons)

My thanks go to the members of Strand II Group 2 for the way in which they took up the challenge to push their boundaries in their discussions—Tim Boerst, Jean-Philippe Georget, Ginger Rhodes, Hannah Bartholomew, Cathy Burns, Barbara Graves, Naomi Robinson, Mike Askew, Paola Stzajn, Eric Roditi, Arthur Powell, Dennis Hembree, Chris Suurtamm, Hanna Haydar, Janete Frant, Sandy Dawson, Axelle Person, John Francisco, and Rose Spanneberg.

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Section 3
Key Issues for Research in the Education and Professional Development of Teachers of Mathematics

To spur research in the field, the section *Key issues for Research in the Education and Professional Development of Teachers of Mathematics* presents the thinking of four key people about major problems of practice and policy, the questions that are crucial to ask, how these might be investigated productively, and what such investigation would take. In this section, Ubiratan D'Ambrosio reflects on the purposes of education and on the role of the mathematics teachers as educators; Edward A. Silver addresses the problem of practice-based professional development for mathematics teachers; and Jill Adler and Barbara Jaworski present a collaborative view on the state of research on mathematics teacher education and how it needs to develop.

Chapter 3.1
Some Reflections on Education, Mathematics, and Mathematics Education

Ubiratan D'Ambrosio, *Universidade Estadual de Campinas, Campinas, SP, BR*

I believe the key problems in the preparation of teachers of mathematics are related to inadequate visions of the purposes of education and of the role of mathematics teachers as educators. Prospective and in-service teachers of mathematics should be always reflecting on changes in education, which result from profound changes in society particularly those in the demographic scenario, in production, in information, and in communication. I will elaborate on the purposes of education and on the role of mathematics teachers as educators.

1. The Goals of Education

I identify a double purpose as to why societies establish educational systems:

- to promote citizenship (which prepares the individual to be integrated and productive in society), which is achieved by transmitting values and showing rights and responsibilities in society; and
- to promote creativity (which leads to progress), which is achieved by helping people to fulfill their potentials and rise to the highest of their capability.

The practice of education is in the present. The major challenge to educators is to manage, in this process, the encounter of the past and of the future, that is, the transmission of values rooted in the past, which leads to citizenship, and the promotion of the new, for an uncertain future, which means creativity. However, in this process, we must be careful. We do not want:

- to transmit docile citizenship—we do not want our students to accept rules and codes which violate human dignity, to be permanently frightened—instead, we want them to assume a critical attitude towards obedience;
- to promote irresponsible creativity—we do not want our students to become bright scientists creating new instruments to increase inequity, arrogance, and bigotry; we want them to be conscious of their acts and of the consequences of their creations.

R. Even, D.L. Ball (eds.), *The Professional Education and Development of Teachers of Mathematics*, DOI 10.1007/978-0-387-09601-8_21,

These are the goals that I hold important in education, hence in mathematics education.

The transmission of values is intrinsic to cultural encounters. The moment of cultural encounters has a very complex dynamic. This encounter occurs between people, as occurred in the conquest and colonization, and between groups. It also occurs in the encounter between the young man or woman who have his or her own culture and the culture of the school with which he or she identifies. The so-called civilizing process, which was carried on by the colonizers—and, we could say, by the school process—is essentially the management of this dynamic.

The promotion of the new must also be part of the school dynamic. Positive results of schooling manifest themselves in the creation of the new. However, regrettably it is frequent to see negative and perverse results which manifest themselves in the exercise of power and the elimination or exclusion of the most creative. This is the essence of the reflections that follow.

Education in this era of science and technology challenges the established approaches "validated" by results in standardized tests. The goals of education go much beyond merely preparing for professional success. Education has a responsibility in building up saner attitudes towards the self, towards society, and towards nature. Indeed, education has the responsibility of furthering creativity.

2. The Role of Mathematics Education

An important component of mathematics education is to reaffirm and, in many cases, to restore the cultural dignity of children. Much of the content of current programs is supported by a tradition alien to the children. On the other hand, children live in a civilization dominated by mathematically based technology and by unprecedented means of information and communication, but schools present an obsolete worldview.

It is equally important to recognize that improving the opportunities for employment is a real expectation that students and parents have of schools. However, preparation for the job market is indeed preparation for dealing with new challenges. There is no point in preparing children for jobs that will probably be extinct when they reach adulthood.[1] To meet the challenges of the new, self-esteem is essential. Self-esteem goes along with cultural dignity.

To acquire cultural dignity and to be prepared for full participation in society requires more than what is offered in traditional curricula. Particularly serious is the situation of mathematics, which is largely obsolete at the present in programs.

[1] For a discussion of labor in the future, see Robert B. Reich, *The work of nations: Preparing ourselves for 21st century capitalism*, Vintage Books, New York, 1992. Harsh views of the future of employment, revealing the inadequacy of current educational systems, can be read in Viviane Forrester, *The economic horror*, Routledge, New York, 1999.

Classroom mathematics has practically nothing to do with the world children are experiencing. To be literate nowadays means much more than reading and writing. Other codes are essential in daily life. Also, proficiency in mathematics means much more than counting, measuring, sorting, comparing, and solving typical problems, aimed at drilling. Even conceding that problem solving, modeling, and projects can be seen in some mathematics classrooms, the main importance is usually given to numeracy, or the manipulation of numbers and operations. Problems and situations, which are present in daily life, are new and unexpected.

I am aware of the fact that these remarks, mainly those in support of ethnomathematics, are interpreted by many as suggesting a reduction of the importance of mathematical contents. This is a grossly mistaken interpretation. We need more and better mathematical contents. Indeed, this means that much of the traditional contents which exhaust current programs should be drastically changed in their presentation. It is a big mistake to consider current mathematics contents in the curricula as something final, essential, and subordinated to criteria of rigor, which are also considered final and essential. Sameness in defining contents redirects innovation to new methodologies aimed at teaching the same, mostly inappropriate, contents. Energy in methodology should be addressed to making advanced mathematics attractive and teachable.

Compromising rigor to the benefit of generating interest and motivation cannot be interpreted as conceptual errors nor as relaxing the importance of serious, advanced mathematical contents in schools.

3. Why Teach Mathematics?

Mathematics is fascinating as a cultural endeavor. It is seen as the imprint of rationality and, indeed, it is the dorsal spine of modern civilization. All the spectacular achievements of science and technology have their basis in mathematics. In addition, the institutions of modern civilization, mainly economics, politics, management, and social order, are rooted in mathematics. It is no surprise that accomplished mathematicians are devoted to mathematics and that the many who tasted mathematics, even without accomplishment, sometimes even with failure, act as fiduciary of mathematics. Administrators, teachers, parents, students, and the population in general see mathematics as the principal subject in schools. Society regards those who do well in mathematics as geniuses. Those who fail are stigmatized.

When looking at mathematics education, we identify two positions: to use education as a strategy for teaching mathematics, defended by those described in the end of the previous paragraph, and to teach mathematics as a strategy for good education.

I like to use a metaphor. Position 1 sees mathematics as the center of the universe. This is the Ptolemaic version of education. Mathematics appears as the absolute

goal. The energy, the Sun (i.e., the children), revolves around the cold and austere focus, the Earth, (i.e., mathematics)![2]

I fully identify with Position 2. As educators, the focus of our mission is on children, young adults, elderly adults, that is, people, who are the source of energy. In this Copernican view, the disciplines, cold and austere, revolve around people, the source of energy.

Is it a good strategy for a good education to have the disciplines, particularly mathematics, revolve around people? I believe so. However, what kind of mathematics?

Bertrand Russell and Albert Einstein said, in the *Pugwash Manifesto* (1955), that a new thinking is needed to achieve equilibrium and safety in a world menaced by war and fear. Is a similar plea to envision a new thinking in mathematics and mathematics education feasible?

4. Mathematics and Mathematics Education in a Changing Civilization

It is widely recognized that mathematics is the most universal mode of thought and that survival with dignity is the most universal problem facing humanity.

We have to look at history and epistemology with a broader view. The denial and exclusion of the cultures of the periphery, so common in the colonial process, still prevail in modern society. The denial of knowledge that affects populations is of the same nature as the denial of knowledge to individuals, particularly children. To propose directions to counteract ingrained practices is the major challenge of educators, particularly mathematics educators. Large sectors of the population do not have access to full citizenship. Some do not have access to the basic needs for survival. This is the situation in most of the world and occurs even in the most developed and richest nations. A new world order is urgently needed. Our hopes for the future depend on learning—critically—the lessons of the past.

When we look at the history of mathematics since the early mathematical manifestations of man, the attempts to compare, classify and organize, measure and count, and infer and conclude much before mathematics was formalized, we recognize mathematical ideas in the confluence of various modes of understanding, such as religion, the arts, the techniques, the sciences, that is, we must assume a transdisciplinarian posture, and we also need to look at all this in different cultural environments, in different traditions, that is, we must assume a transcultural posture.

With respect to cognition, history shows us that the emergence of modern science is closely associated with the recognition of an exclusive rational dimension of thinking. Recently, there has been acknowledgement of other dimensions in the capacity of reasoning and understanding. Multiple intelligences, emotional

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[2] According to Bertrand Russell, "Mathematics possesses not only truth, but supreme beauty—a beauty cold and austere, like that of sculpture".

intelligence, spiritual intelligence, and numerous approaches to rationality have important consequences for education. Also, mental tasks performed by individual human beings are better understood thanks to the advances of artificial intelligence. For mathematics education, these advances strongly challenge the concepts of skill and drilling.

The enormous changes in society, particularly due to demographic dynamics, raise to unbearable levels the exclusion of large sectors of the population, both in developed and undeveloped nations. The same among nations which as a consequence of globalization are facing a tendency toward federation. The exclusion of countries of the benefits of progress and advancement is unsustainable. An explanation for the current perverse concept of civilization asks for a deep reflection on colonialism. This is not to place blame on one or another; this is not an attempt to redo the past. Rather, it is the moment to understand the past as a step to move into the future. To accept inequity, arrogance, and bigotry is irrational and may lead to disaster.

Since mathematics has everything to do with this state of the world, its autonomy in the curriculum and its central role as the dominating discipline and as an educational sphere in itself should be reconsidered. To paraphrase Mikhael Gromov (1998), we shall need for this the creation of a new breed of mathematical teachers able to mediate between mathematics and the other disciplines. However, current curricula, in all levels of education, look like a selection of non-overlapping sets. As a result, there is a lack of equilibrium between mathematical competence and a broader vision of the world and of society among teachers.

Curriculum is the strategy for educational action. An answer to my criticism of the lack of equilibrium is my proposal of literacy, matheracy, and technoracy (1999). It is a proposal for a curriculum based on developing a broad perception of the complexity of the world and of society and providing the instruments to deal with such a complexity. Literacy is the capability of processing information, such as the use of written and spoken language, signs and gestures, and codes and numbers. Nowadays, reading includes the competency of numeracy, the interpretation of graphs and tables, and other ways of informing the individual. Reading even includes the understanding of the condensed language of codes. These competencies have much more to do with screens and buttons than with pencil and paper. Matheracy is the capability of inferring, proposing hypotheses, and drawing conclusions from data. It is a first step towards an intellectual posture, which is almost completely absent in our school systems. Matheracy is closer to the way mathematics was present both in classical Greece and in indigenous cultures. The concern goes far beyond counting and measuring. Matheracy proposes a deep reflection about man and society. This is the central idea behind the origins of mathematics. It should not be restricted to an elite, as it has been in the past, but should be conveyed by education. Technoracy is the critical familiarity with technology. Of course, the operative aspects of it are, in most cases, inaccessible to the lay individual. However, the basic ideas behind technological devices, their possibilities and dangers, and the morality supporting the use of technology are essential issues to be raised among children at a very early age. History shows us that ethics and values are intimately related to technological progress.

The three constitute what is essential for citizenship in a world moving swiftly towards a planetary civilization.

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Chapter 3.2
Toward a More Complete Understanding of Practice-Based Professional Development for Mathematics Teachers

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Teacher educators, professional developers, and researchers have recently shown great interest in the design and facilitation of an approach to mathematics teacher education that is commonly called practice-based professional development (PBPD). At the conceptual and operational heart of PBPD one finds professional learning tasks (PLTs)—activities that are situated in and organized around components and artifacts of instructional practice that replicate or resemble the work of teaching. PLTs are often built around artifacts of practice such as curriculum materials, video or narrative records of classroom teaching episodes, and samples of student work. In particular, video and narrative cases (e.g., Smith, Silver, & Stein, 2005) have been extensively used in PBPD.

PLTs provide stimuli and opportunities for teachers to develop and refine the knowledge needed in pedagogical practice (Ball & Cohen, 1999). PLTs make the work of teaching (e.g., designing, preparing to teach, or enacting classroom lessons or larger units of instruction; analyzing evidence of student thinking from verbal statements or written products) available for investigation and inquiry. A common underlying characteristic of PLTs is that they provoke teachers to treat a particular situation as problematic. Helping teachers discern the problematic component of the task and consequently own it and see it as meaningful is a critical aspect of the work that goes in the design and facilitation of PLTs. The highly contextualized nature of these tasks is intended to allow teachers to propose, debate, and consider solutions to pedagogical dilemmas and explore pedagogical possibilities as they move back and forth between past and current teaching experiences and the activity space of the professional development experience (see **Fig. 3.2.1**).

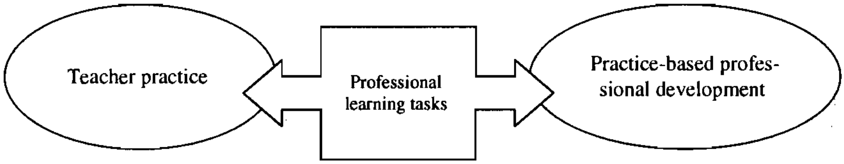

Fig. 3.2.1 The role of professional learning tasks

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Many scholars have pointed to the potential benefits of having teachers learn in and through professional practice (e.g., Ball & Bass, 2003; Ball & Cohen, 1999; Lampert, 2001; Smith, 2001; Stein, Smith, Henningsen, & Silver, 2000). At this time the theoretical development of ideas related to PBPD and PLTs has far outstripped the empirical evidence base. The time is ripe for empirical investigations of practice-based professional development that can contribute to the currently impoverished evidence base.

Two distinctive features of PBPD lie at the core of claims regarding its anticipated efficacy:

1. Unlike conventional approaches to professional development, PBPD treats mathematics content, mathematics pedagogy, and student thinking in an integrated manner. Advocates argue that this treatment of knowledge domains is similar to the way that they appear in the actual work of mathematics teaching, thereby increasing the likelihood that teachers will acquire knowledge that is useful and usable in their practice (Smith, 2001).
2. PBPD learning experiences are highly connected to and contextualized in professional practice settings, and advocates for this approach argue that this results in useful and usable knowledge that builds mathematics teachers' capacity for the kinds of complex, nuanced judgments required in mathematics teaching (Ball & Bass, 2003).

One can generate a number of questions that could stimulate productive research inquiry by considering critically each of these distinctive features of PBPD.

When teachers engage in PLTs that deliberately entangle aspects of mathematics content, pedagogy, and student learning, we can ask, "What are they learning?" and "How do we know?" Despite the general appeal and hypothesized benefits of learning in and from practice through engagement with PLTs, little is known about if and how teachers actually learn mathematics or pedagogy in such settings. How does PBPD provide opportunities for teachers to acquire mathematics knowledge for teaching? How does PBPD provide opportunities for teachers to become more pedagogically proficient? What characteristics of PLTs (e.g., task design), and associated features of professional development (e.g., facilitation), appear to support or inhibit teachers' learning of mathematics or pedagogy? What forms of evidence are adequate to make claims about such learning?

Consider, for example, the matter of teachers learning mathematics through PBPD. Ball and Bass (2003) argue persuasively that "...mathematical knowledge for teaching has features that are rooted in the mathematical demands of teaching itself" (p. 4). Thus, we can hypothesize that PBPD using PLTs can develop the mathematical knowledge that teachers need in their instructional practice. Because professional learning tasks simulate and emulate the demands of teaching practice, the strong potential benefits of knowledge gained in this way are clear. Thus, PLTs may be particularly well suited to serve as tools to assist teachers to learn to use mathematics in the ways they will do so in practice—such as interpreting, making mathematical and pedagogical judgments about, and formulating useful responses to students' questions, solutions, difficulties and insights—as well as to serve as

activities that strengthen and deepen their understanding of important mathematical ideas. The theoretical basis for such claims is clear, but empirical evidence is lacking to support, refute. or modify the claims.

Turning to the second distinctive feature of PBPD—its highly contextualized nature—we can identify additional questions for research. How do teachers move from the particularities associated with specific PLTs to more general ideas, understandings, or principles that can be applied to instructional settings or demands that differ from those in the PLTs? What characteristics of PLTs, and associated features of professional development (e.g., facilitation), appear to support or inhibit teachers' application of knowledge from PBPD settings to their own instructional practice? What forms of evidence are adequate to make claims about the usefulness and usability of knowledge gained by teachers in PBPD settings? Investigations provoked by these and similar questions should help the field make progress—not only allowing refinement of the theoretical basis for PBPD but also stimulating practical improvements in this very promising approach to teacher education.

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Chapter 3.3
Public Writing in the Field of Mathematics Teacher Education

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and **Barbara Jaworski**, *Loughborough University, Loughborough, UK*

Mathematics teacher education can be seen as directly related to activity in mathematics classrooms and the success (or other) of students learning mathematics worldwide. In what ways does what is published in the field of mathematics teacher education inform us about key question and issues, about programmes for educating teachers, and about research findings? We refer specifically to an International Congress on Mathematical Education (ICME) survey (with Adler as chair) and to the *Journal of Mathematics Teacher Education* (JMTE), the leading journal in the field (with Jaworski as editor in chief).

1. Defining the Scope and Nature of the Field: an Icme 10 Survey

In July 2004, an international team of five mathematics educators and researchers presented the results of their survey of research in mathematics teacher education from 1999 to 2003, during a plenary session at ICME 10, in Copenhagen. The details of the survey have since been published in *Educational Studies in Mathematics* (November 2005), and the authors conclude that the survey provides a vantage point from which to reflect on the current state of the field of mathematics teacher education research.

Briefly, the survey included published research in international mathematics education journals, international handbooks of mathematics education, and international mathematics education conference proceedings. Some regional sources from various parts of the world were also included. The investigation focused on who was writing, from and in what settings, with what theoretical frameworks, and with what sorts of study designs for what core questions. The range of findings and conclusions produced in these studies were also examined. Four themes stood out from the initial investigation of almost 300 published papers. These themes were then systematically elaborated through a focused study of a 160 papers across two key journals in the field (*JMTE* and the *Journal for Research in Mathematics Education* [JRME]) and a key set of conference proceedings (*Psychology of Mathematics Education* [PME]). Four substantive claims were made, evidenced, and commented on from different perspectives. Here we summarise rather than debate these claims.

R. Even, D.L. Ball (eds.), *The Professional Education and Development of Teachers of Mathematics*, DOI 10.1007/978-0-387-09601-8_23,

Claim #1: Small-scale qualitative research predominates. The authors clarify that by small-scale qualitative research they include research that focuses on a single teacher or on small groups of teachers (n<20) within individual programmes or courses. They explain that the systematic analysis of the 160 papers referred previously revealed that there were fewer than 20 teachers in close to 70% of the studies reported. In short, a significant percentage of papers surveyed were small case studies.

Claim #2: Most teacher education research is conducted by teacher educators studying the teachers with whom they are working. In addition to most studies being small case studies, the survey also revealed the phenomenon of what some would call "insider" research—where researchers have some direct involvement and thus some interest in the case being studied. Of articles representing research that focuses on teacher education between 1999 and 2003, 90% of *JMTE*, 82% of *PME*, and 72% of *JRME* articles were of this type.[1]

Claim #3: Research in countries where English is the national language dominates the literature surveyed. The following figures were presented to substantiate this claim. In *JMTE* between 1998 and 2003, 80% of the articles are from such countries. In *JRME* this figure is 71%. It is less stark, but nevertheless prevalent, in *PME* between 1999 and 2003, when the percentage is 43%. One effect posited was that questions that come to constitute the research field are driven by concerns in particular contexts and thus might not reflect the diversity of problems in teacher education that exist globally. This was a controversial and contested claim, both at the ICME 10 Congress after the presentation, as well as during discussion after our presentation at the 15th ICMI study conference. The objection was that this is self-evidently skewed by the journals and conference proceedings focused upon, as these were English-language journals. We will not take the debate further here but rather ask: are the questions that drive mathematics teacher education research appropriate across diverse cultural contexts and conditions?

Claim #4: Some questions have been studied extensively, while other important questions remain unexamined. The survey noted that much of the research, particularly in the United States, was concerned with reform and involved efforts to show that particular programmes of teacher education "work". As a field, we are more informed about teachers learning or relearning mathematics, teachers learning about students' thinking, their language, their orientations, and pedagogical practices. As a consequence of the focus on reform, however, we know much less than we should about teachers' learning from experience: what they learn, whether they learn, and what supports learning from experience. We also know too little about teachers' learning to directly address inequality and diversity within their teaching of mathematics, and we lack comparisons in the field of different opportunities to learn. Finally, we have done much less studying of what it means to scale up a programme or extend a programme that has worked in one setting to another setting.

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[1] As there were only seven *JRME* papers between 1999 and 2003 that fit our survey, this percentage can only be regarded as a very rough measure.

2. The Journal of Mathematics Teacher Education

It seems clear from reports from the survey that *JMTE* is an important publishing resource in our area. Its mission statement reads as follows:

> The Journal of Mathematics Teacher Education is devoted to research into the education of mathematics teachers and development of teaching that promotes students' successful learning of mathematics.
>
> JMTE focuses on all stages of professional development of mathematics teachers and teacher educators and serves as a forum for considering institutional, societal and cultural influences that impact on teachers' learning, and ultimately that of their students.
>
> Critical analyses of particular programmes, development initiatives, technology, assessment, teaching diverse populations and policy matters, as these topics relate to the main focuses of the journal, are welcome.

JMTE is a young journal: at the time of this writing, the tenth volume had just been completed. The journal has an acceptance rate of 18% for research articles. The contents of *JMTE* are compiled mainly from submitted articles in two categories: accounts of relevant research and accounts of teacher education programmes around the world. The latter has been established to encourage publication from a wide range of countries. However, papers come mainly from the developed world, with a high proportion (>50%) from North America. Nevertheless, the journal invites papers from all countries and works hard to help non-English-speaking authors complete a paper in English. In addition, *JMTE* publishes special issues, either compiled from submitted papers that centre around one important topic area (an example was "community" in *JMTE* 6, 3) or a topic area proposed by a prospective guest editor and accepted by the editorial team (e.g., "Relations between theory and practice", *JMTE* 9, 2). Most recently, a special triple issue (*JMTE* 10, 4–6) was completed, focusing on the nature and role of tasks in mathematics teacher education.

In accordance with the survey, most research articles report small-scale qualitative research that comes largely from teacher educators researching their own practice. From the volumes so far we see evidence of a developing field from papers in which research provides evidence of individual teacher or small teacher group development within a particular programme;

- that a learning community exists or is developed;
- that teachers engage in critical inquiry, reflective practice, or action research;
- that a teacher education programme links closely with the practice field;
- of teachers and teacher educators working side by side in and out of school; and
- that teachers or student teachers learn from engagement in research.

In all cases there is evidence of deep learning and changes to practice. It is clear that such research both documents learning in practice and, in many cases, contributes to that learning. Editors and reviewers look for a suitably critical stance from authors reporting research into their own practices or programmes. Nevertheless, we should ask what endures and grows from these published accounts. What can take the field beyond the local and special-case nature of such research? How is it possible to generalise from such studies? What methodologies will provide larger-scale

evidence of teacher learning and developmental approaches that result in better teaching and learning? What theory can we see emerging from research in the field? There is a wide range of theoretical models or frameworks for developmental practice or to explain or analyse teacher and teaching development. However, there are, as yet, no "grand" theories to compare, for example, with theories of learning, such as constructivism or sociocultural theories. Indeed, attempts to distil guidelines for practice from learning theory have resulted in pseudo-theoretical appellations ("constructivist teaching" is one common example), which have no substance or credibility in the practical world.

Rightly, the world of practice expects more from research than can be seen currently; however, the nature and prevailing conditions of and for research militate against fulfilment of such expectations. Political short-termism, local and national, perpetuates the status quo: teacher educators are required to publish; large research teams are difficult to convene and fund; longitudinal studies are both expensive and, crossing different administrations, not always politically compatible. Developmental sustainability beyond the end of a project is accordingly difficult to enable. However, there are deeper issues in the field that we have to consider before changes in policy can be expected to change the developmental landscape.

3. Research Programmes

In response to the previous discussion, we end with a focus on two areas of research that are starting to address some of the key issues raised.

3a. Mathematics for Teaching

An interesting observation from the survey and an overview of *JMTE* is that in the current foci in mathematics teacher education research, the specificities of mathematics recede. Here we bring mathematics back into focus through a discussion of what elements of a research programme will take forward the field of mathematics teacher education research.

What mathematics is selected into mathematics teacher education courses and programmes, be these mathematic courses, or mathematics methods courses? How is this mathematics taught and evaluated and with what effects on teachers' (both prospective and practising) learning mathematics and mathematical know-how pertinent to the demands of teaching? Generally referred to as mathematics for teaching, there is now a growing interest in describing the specificity of the ways teachers need to know and be able to use mathematics effectively in their teaching and the opportunities teachers are provided for learning this situated or professional knowledge. There is a growing appreciation that this kind of mathematical focus and learning is left to the vicissitudes of practice. Just as we know that in school there are gaps between curriculum intentions, implementation, and attainment, we need to

acknowledge that even in programmes where there is a focus on what is becoming valued as mathematics for teaching, we need empirical studies on such, what comes to be learned, by whom, and with what effects. One such study (the QUANTUM research project), in South Africa, for example, has revealed a range of pedagogic modalities at work through a cross-case analysis of different sites of formalised in-service programmes. The ways in which mathematics and pedagogy are integrated differ across courses and provide different (and potentially inequitable) opportunities for learning mathematics for teaching (Davis, Adler, & Parker, 2007). Moreover, through a focus on assessment that mathematics courses specifically designed for formalised in-service teachers in these programmes rarely required teachers in these courses to demonstrate competence in reasoning mathematically neither in relation to a particular mathematical idea or concept nor in relation to how this might be done and so responded to by the teacher (Adler & Davis, 2006). We need to know a great deal more about the kinds of mathematical learning opportunities afforded in both formal and informal sites of teacher education so as to be able to improve the quality of teacher education, particularly in relation to what and how mathematics is selected, taught, and assessed.

3b. Research Partnerships Between Teachers and Educators

The survey and *JMTE* experience show the scarcity of long-term research programmes in which development can be studied. Setting this alongside a scarcity of research linking teacher education to the learning of mathematics in classrooms, a study in Norway, currently in its fifth year, offers potential significance. Here, teachers and educators (didacticians) work together to study development of mathematical learning activity in classrooms through the creation of inquiry communities. Both groups bring important knowledge and experience to the research interface that forms the basis of community; both engage in inquiry to introduce and explore innovative practices and challenge traditional classroom approaches. Original funding for four years from the Research Council of Norway has been extended to four more years, and the original team in one city and university has extended to five locations in Norway. At the end of the first four years, findings show significant development for individual teachers or groups of teachers in project schools and clear evidence of pupils' engagement in practices that motivate students and foster mathematical understanding. The locus of power in the early years has rested with didacticians, teachers taking time to find a voice and influence the directions of activity. Institutional and sociocultural factors also have dominated practices for teachers, often working against preferred practices within the project. More recently, schools have sought and attracted their own funding, and school leaders, together with didacticians, design activities and take responsibility for their operationalisation in institutional settings. In each of the five locations we see substantial teams of didacticians and a range of participating schools. Funding from the research council is matched by local funding from a range of sources. The scale of this research and the potential

it creates for development is a result of ambitious design, adequate (although not generous) funding, a sincere will to develop partnerships with shared power and responsibility and a long-term vision. In these respects the Norwegian research is addressing several of the concerns reported previously (Jaworski et al. 2007).

There seems to be a necessity for seeking out and reporting from projects that are starting to address the issues raised, particularly those with large-scale and long-term funding, as a basis for encouraging this longer-term vision.

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Strengthening Practice in and Research on the Professional Education and Development of Teachers of Mathematics: Next Steps

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This Study brought together 147 scholars and practitioners from 35 countries to discuss the professional formation of teachers of mathematics. Their individual contributions have been assembled to create a volume filled with descriptions of programs and projects, as well as concepts and data. Readers interested in the development of teachers of mathematics will find in this book many ideas and people relevant to their own work, much as was discovered by those who participated in the Study. The conference was lively and intense as ideas and people from around the world interacted around this fundamental problem: Mathematics education works virtually nowhere as well as it needs to if we are to prepare students for life in world where practical, intellectual, and critical quantitative competence will matter more than ever before, and for more people. Although our systems, resources, and results differ, no country is satisfied with the quality or the reach of contemporary mathematics education.

This Study was founded on the premise that teachers are central to the improvement of mathematics education, a premise that should be obvious, but too often is not. Since what students learn is a function of the opportunities they have and how those opportunities are managed, improvements that aim solely on curriculum or standards are unlikely to make the sorts of impact on students' learning that some assume. Despite the importance of teachers, however, the approaches to their education often do not help them develop the skills and insights needed for practice. This Study began an international conversation about what we do and the questions we have. Equally important will be the next steps that follow. We suggest below three main problems that could profit from stronger and more systematic international connections focused on improving the education and professional development of teachers.

First is *the need to focus teachers' education on practice*—and the problem of doing it effectively. This was a focus of the Study—to learn what is done around the world, and what the challenges are. On one hand, it should be obvious that teaching is a practice and that, therefore, teachers' education must provide sustained, systemic opportunities for teachers to learn and develop their effectiveness with

that practice—with the complex tasks of their work. Many Study participants provided examples of teachers working in and on practice to develop their knowledge and skill. In some cases, the practice on which teachers worked was their own. In others, teachers studied cases, watched videotapes, or examined students' work. In still others, they co-planned lessons and observed the enactment of lessons across classrooms with different pupils. On the other hand, lacking is a shared articulation of a "curriculum" that would underlie teachers' opportunities to learn in and from their practice. Often mentioned was that teachers need time to work on practice with colleagues. However, although time to meet is important, time is not enough. As important is more specificity about how practice can be harnessed for teachers' learning and what is important to learn in and from practice. The international community could work to develop ways to support teachers learning in and from practice, through making this a focus of discussion, collegial exchanges, and a topic at international meetings.

One crucial agenda for future work is that, for a focus on practice to be more consistently possible, teachers need robust examples with which to work, either from their own classrooms, or collected systematically from others. To be useful, artifacts of practice require careful collection and development. This is an area in great need of support if teachers' professional learning–as that of other professionals— is to be grounded in problems and cases of professional practice. Study presentations included rich examples of cases, and other "captures" of practice for teachers' observation and study. But no system exists for considering what makes an example robust enough to use effectively. No scheme was offered for typologies of cases, nor how to compare different media—written cases, student work, videos, to name a few.

Another challenge is the need to develop approaches to *teaching* practice. Although video can provide a concrete and vivid resource for professional study, unfocused viewing is like reading a text without a purpose. Productive study of classroom teaching depends on questions that frame the work. What are the features of tasks that support teachers' investigation and learning of practice?

A second significant issue on which the international community could focus and build collective capacity is the identification and *development of teacher developers*. In different countries, a wide range of professionals is responsible for supporting teachers' learning, and yet often these individuals have little preparation for their work. Inconsistent is whether or not they have taught, studied mathematics in depth, or have substantial insight into students' thinking. We learned of projects that directly support the development of teacher leaders, but around the globe, there is little rule about what qualifies someone to take on the role of teacher developer. Moreover, there is little support for their ongoing learning.

There is, to begin with, a problem of conceptual diffusion. No single word or phrase exists to describe the professionals who work with teachers: two- and four-year college teachers of mathematics; university mathematicians; university mathematics educators; district and school personnel responsible for the pre-service and in-service education of teachers of mathematics; doctoral students preparing to become mathematicians or mathematics educators in colleges and universities; and

professional developers who contract with schools or districts to provide workshops, institutes, or other programs for K-12 teachers of mathematics. In addition, there is no clear role group or identity. Neither university mathematicians nor faculty members who teach content or methods courses in mathematics for prospective teachers ordinarily think of themselves as "professional developers"—and yet they are. Consider that in both sorts of courses, prospective teachers explicitly learn disciplinary knowledge that forms the basis of the content they will teach to students. They also develop ideas about how that content is taught. The same can be said of school-based teacher developers who help teachers learn the content of new curricula, improve their skills, or study new topics. These different sorts of professionals have rarely been considered collectively as "teacher developers" or "teacher educators." The lack of focus on these people weakens international efforts to improve teacher education and professional development.

Because we do not think of them as a professional group, there has been little work on what "teachers of teachers" have to know and be able to do to support teachers' learning. What mathematical knowledge is needed for this work? How do they have to be able to use practice as a resource for teachers' learning? What is there to learn about the design and delivery of particular approaches to the professional development of teachers of mathematics? These questions are complicated in different ways by the professional background of different types of teacher educators. What mathematicians might need to learn to be effective with teachers is likely different from the learning needs of teachers who move into leadership roles. What might be common and what is special?

Programs of and approaches to supporting teacher developers' learning could be better shared across nations. The improvement of teacher education depends on the quality of those responsible for its delivery; much could be gained through greater exchange of ideas and approaches.

Third, with the increasing demand for evidence of results, a growing need exists for *valid and reliable assessments of teachers' learning*. Again, whereas some Study participants provided insight into the outcomes of particular programs, the international community has not mobilized collective effort to build useful tools and methods for teacher assessment. In an era of increasing accountability, relying on teacher-learners' reflections on their learning is inadequate. Yet, given the need to improve teachers' practice, tests of pure mathematical content knowledge are also insufficient. Crucial are "measures" and other means of the quality and effectiveness of teachers' mathematics instruction. Such assessments are important to assess the outcomes of professional education and to compare the effects of different approaches. With greater capacity to do this, the international community would build knowledge about how to make teachers' education more effective. Measures and assessments of instructional practice are also important to be able to investigate the effects of teaching on students' learning, and to trace the complex connections between teachers' knowledge and skill and their students' growth.

We need to know more about ways to assess practice and the learning of practice. And we need to understand what aspects of such assessment are culturally specific and which are internationally shareable. How does the fact that teaching is a cultural

activity (Stigler, & Hiebert, 1999) affect efforts to assess it? Cross-cultural work on the assessment of practice could contribute both to our understanding of how deeply different teaching is or is not, and what the specific and crucial differences are. Such cross-national work would also elaborate our sense of teaching as practice across cultures. In different countries, the political environments, societal and cultural factors, and the place of formal schooling and therefore of school professionals, varies. Additionally, curricular differences, variation in teachers' responsibilities, and differences in the pupils they teach, shape practice. Still only partly understood is the depth of these differences in terms of the demands and nature of teachers' practice. This area of assessment is of urgent need, especially in countries where the policy environment is threatening to overwhelm professional judgment and expertise with political structure. If mathematics education professionals do not seize the lead on issues of rigorous assessment of practice, others will, and their perspectives on this will likely dominate. This is a time to assert that teaching practice can be assessed validly, and that teaching requires professional knowledge and skill. To argue otherwise is to rescind authority for our own profession, and crucial levers for its improvement.

These three areas were ones that participants spoke about and that the Study included. Yet, quite clear is that these are also areas for fruitful further work. To take advantage of the active interest in teacher development and teacher learning and the fascinating differences within and across countries, the international community could mobilize to focus more on teachers' opportunities to learn. In particular, we see great potential to focus sharply and deliberately on ways to center such opportunities on learning in and for practice; to investigate and strengthen the preparation and support of "teacher developers"; and to work together to build professionally valid and reliable approaches to assessing teachers' learning and the practice they are able to deliver as a result.

How might more opportunities for international and cross-cultural work on teacher education and professional development be supported? One is clearly in the creation of more international meetings and conferences focused on teacher formation. A second lies in publishing, in journals and books, as well as in electronic web-based forms. Such publishing needs to include the publishing of artifacts and tools as well as descriptions and results. But a third approach is to make teachers' education and development a more regular part of other mathematics education conferences, meetings, projects, and programs. Often we organize programs focused centrally on curriculum content and on student learning, and although these are at the heart of mathematics education, too often we overlook the role of teachers and their capacities, or the key role of skillful instruction in students' learning. At times we lapse into discussions that make student learning seem to arise entirely on its own, or by design, without nuanced structure, guidance, or interaction. A greater emphasis is needed on understanding the connected demands on teaching practice and the consequent requirements for professional education. This Study demonstrated the enormous potential of the international community to build capacity in the practice of teacher education, teachers' formation, and the study of both. Instruction is central to students' learning of mathematics; thus, how teachers

and their instructors are prepared, and the tools and methods we have for studying alternative approaches, are also central. The most important outcome of this, the first ICMI study conference on teacher development around the world, is to install the education and continuing development of teachers of mathematics as a central problem of mathematics education, rather than as a domain of casual exchange and a parenthesis to the "main" work of our field.

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Subject Index

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